Continuous Galerkin methods for solving Maxwell equations in 3D

Patrick Ciarlet, Erell Jamelot

Patrick.Ciarlet@ensta.fr

POEMS, UMR 2706 CNRS-ENSTA-INRIA

Time-dependent Maxwell equations

- *In vacuum, over the time interval* $]0, T[$, $T > 0$.
  Find $(\mathcal{E}(t), \mathcal{H}(t)) \in \mathbf{L}^2(\cdot) \times \mathbf{L}^2(\cdot)$ such that

$$\begin{cases} \varepsilon_0 \partial_t \mathcal{E} - \mathbf{curl}\, \mathcal{H} = -\mathcal{J} \ ; \\ \mu_0 \partial_t \mathcal{H} + \mathbf{curl}\, \mathcal{E} = 0 \ ; \\ \mathrm{div}\,(\varepsilon_0 \mathcal{E}) = \rho \ ; \\ \mathrm{div}\,(\mu_0 \mathcal{H}) = 0 \ ; \\ \mathcal{E}(0) = \mathcal{E}_0 \ , \ \mathcal{H}(0) = \mathcal{H}_0 \ . \end{cases}$$

$$\Big(\partial_t \mathcal{J} \in L^2(0, T; \mathbf{L}^2(\cdot)),\ \rho \in \mathcal{C}^0(0, T; L^2(\cdot)) \ ; \partial_t \rho + \mathrm{div}\, \mathcal{J} = 0.$$

$$\mathcal{E}_0 \in \mathbf{H}(\mathbf{curl}\,, \cdot),\ \mathrm{div}\, \mathcal{E}_0 = \frac{1}{\varepsilon_0} \rho(0) \ ; \mathcal{H}_0 \in \mathbf{H}(\mathbf{curl}\,, \cdot),\ \mathrm{div}\, \mathcal{H}_0 = 0. \Big)$$

- Goal: compute the EM field around a perfect conducting body $\mathcal{O}$, with Lipschitz polyhedral boundary.

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Time-dependent Maxwell equations (2)

But... Consider a *bounded* computational domain Ω, with Lipschitz polyhedral boundary.

Its boundary $\partial\Omega$ is split as $\partial\Omega = \bar{\Gamma}_C \cup \bar{\Gamma}_A$, with $\bar{\Gamma}_C = \partial\mathcal{O} \cap \partial\Omega$.
A *Silver-Müller* boundary condition is imposed on the artificial boundary Γ_A: incoming plane waves ($\mathbf{e}^\star \neq 0$), or 1st order absorbing condition ($\mathbf{e}^\star = 0$).

- *Boundary conditions*

$$\begin{cases} \mathcal{E} \times \mathbf{n} = 0 \text{ on } \Gamma_C \ ; \\ (\mathcal{E} - \sqrt{\dfrac{\mu_0}{\varepsilon_0}}\mathcal{H} \times \mathbf{n}) \times \mathbf{n} = \vec{\mathbf{e}}^\star \times \mathbf{n} \text{ on } \Gamma_A . \end{cases}$$

$$\left(\partial_t \vec{\mathbf{e}}^\star \in L^2(0,T;\mathbf{L}^2(\Gamma_A)). \right)$$

- Consequences: some *"additional" boundary conditions*

$$\mathcal{H} \cdot \mathbf{n} = \mathcal{H}_0 \cdot \mathbf{n} \ ; \ (\mathbf{curl}\,\mathcal{H}) \times \mathbf{n} = \mathcal{J} \times \mathbf{n} \text{ on } \Gamma_C .$$

$$\left. \begin{array}{l} (\mathbf{curl}\,\mathcal{E}) \times \mathbf{n} = \dfrac{1}{c}\partial_t \mathbf{e}^\star_T - \dfrac{1}{c}\partial_t \mathcal{E}_T \\ (\mathbf{curl}\,\mathcal{H}) \times \mathbf{n} = \mathcal{J} \times \mathbf{n} + \varepsilon_0 \partial_t (\mathbf{e}^\star \times \mathbf{n}) - \dfrac{1}{c}\,\partial_t \mathcal{H}_T \end{array} \right\} \text{ on } \Gamma_A .$$

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Time-dependent Maxwell equations (3)

2nd order in time, electric field $\mathcal{E}$...

- Equation

$$\left\{\begin{array}{l} \partial^2_{tt}\mathcal{E} + c^2\,\mathbf{curl}\,\mathbf{curl}\,\mathcal{E} = -\dfrac{1}{\varepsilon_0}\partial_t\mathcal{J}\ ;\ \partial_t\mathcal{E}(0) = \mathcal{E}_1 \\ \left(\mathcal{E}_1 := \dfrac{1}{\varepsilon_0}\Big(\mathbf{curl}\,\mathcal{H}_0 - \mathcal{J}(0)\Big)\right) \end{array}\right. .$$

- Functional space (see for instance [Ben Belgacem-Bernardi'99])

$$\mathcal{T}_E := \{\mathbf{v} \in \mathbf{H}(\mathbf{curl}\,,\Omega)\ :\ \mathbf{v}\times\mathbf{n}_{|\partial\Omega} \in \mathbf{L}^2_t(\partial\Omega),\ \mathbf{v}\times\mathbf{n}_{|\Gamma_C} = 0\}.$$

- Variational Formulation

  Find $\mathcal{E}(t) \in \mathcal{T}_E$ such that

(1)
$$\left\{\begin{array}{l} <\partial^2_{tt}\mathcal{E},\mathbf{v}> + c^2(\mathbf{curl}\,\mathcal{E},\mathbf{curl}\,\mathbf{v})_0 + c\frac{d}{dt}(\mathcal{E}_T,\mathbf{v}_T)_{0,\Gamma_A} \\ \qquad\qquad = -\frac{1}{\varepsilon_0}(\partial_t\mathcal{J},\mathbf{v})_0 + c\frac{d}{dt}(\vec{\mathbf{e}}^\star_T,\mathbf{v}_T)_{0,\Gamma_A},\ \forall\mathbf{v}\in\mathcal{T}_E. \end{array}\right.$$

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"Electrostatic" model

(see [Jr'05] for proofs.)

- Set of static equations

$$\left\{\begin{array}{l}\text{Find } \mathcal{E} \in \mathbf{L}^2(\Omega) \text{ such that} \\ \mathbf{curl}\,\mathcal{E} = \mathbf{f},\ \operatorname{div}\mathcal{E} = g \text{ in } \Omega\ ;\ \mathcal{E} \times \mathbf{n}_{|\partial\Omega} = 0.\end{array}\right.$$

$\Big($ $\mathbf{f} \in \mathbf{L}^2(\Omega)$, $g \in L^2(\Omega)$: $\operatorname{div}\mathbf{f} = 0$ and $\mathbf{f} \cdot \mathbf{n}_{|\partial\Omega} = 0.$ $\Big)$

- *Define* $\mathcal{X}_E^0 := \mathbf{H}_0(\mathbf{curl}\,, \Omega) \cap \mathbf{H}(\operatorname{div}\,, \Omega)$.
- Hypothesis: the semi-norm associated to

$$(\cdot,\cdot)_{X^0} :\ (\mathbf{u},\mathbf{v}) \mapsto (\mathbf{curl}\,\mathbf{u}, \mathbf{curl}\,\mathbf{v})_0 + (\operatorname{div}\mathbf{u}, \operatorname{div}\mathbf{v})_0$$

is a norm on $\mathcal{X}_E^0$, which is *equivalent* to the full norm.
From [Weber'80], [Fernandes-Gilardi'97], [Amrouche-Bernardi-Dauge-Girault'98]:
Assume for instance that $\partial\Omega$ *is connected*.

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"Electrostatic" model (2)

- *Define the variational problem* (P^0) *in* $\mathcal{X}_E^0$

$$\begin{cases} \text{Find } \mathcal{E} \in \mathcal{X}_E^0 \text{ such that} \\ (\mathcal{E}, \mathbf{v})_{X^0} = (\mathbf{f}, \mathbf{curl}\, \mathbf{v})_0 + (g, \operatorname{div} \mathbf{v})_0, \quad \forall \mathbf{v} \in \mathcal{X}_E^0. \end{cases}$$

- Theorem: $\exists! \mathcal{E} \in \mathcal{X}_E^0$ *solution to problem* (P^0).
  In addition, $\mathcal{E}$ *is the only solution to the electrostatic model.*

- Hypothesis: the subspace of regular fields $\mathcal{X}_{E,R}^0 := \mathbf{H}^1(\Omega) \cap \mathcal{X}_E^0$ is *dense* in $\mathcal{X}_E^0$.
  From [Grisvard'85] + [Birman-Solomyak'87]: ok if Ω *is convex*.
- Conclusion: numerical approximation with the Continuous P_k Lagrange FE is possible.

- With respect to (1), (P^0) is an Augmented Variational Formulation (AVF).
- Similar AVF and results for $\mathcal{H}$ in $\mathcal{X}_H^0 := \mathbf{H}(\mathbf{curl}\,, \Omega) \cap \mathbf{H}_0(\operatorname{div}\,, \Omega)$ (with $(\cdot, \cdot)_{X^0}$)...

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"Electrostatic" model (2 proof)

- *Define the variational problem* (P^0) *in* $\mathcal{X}_E^0$

$$\begin{cases} \text{Find } \mathcal{E} \in \mathcal{X}_E^0 \text{ such that} \\ (\mathcal{E}, \mathbf{v})_{X^0} = (\mathbf{f}, \mathbf{curl}\,\mathbf{v})_0 + (g, \operatorname{div}\mathbf{v})_0, \quad \forall \mathbf{v} \in \mathcal{X}_E^0. \end{cases}$$

- Theorem: $\exists ! \mathcal{E} \in \mathcal{X}_E^0$ *solution to problem* (P^0)*.*
In addition, $\mathcal{E}$ *is the only solution to the electrostatic model.*

Proof:

(i) Existence and uniqueness of the solution to problem (P^0) is straightforward.

(ii) $\forall g' \in L^2(\Omega)$: $\exists ! \phi \in H_0^1(\Omega)$ such that $\Delta\phi = g'$.
As $\mathbf{v} = \nabla\phi \in \mathcal{X}_E^0$, there holds $(\operatorname{div}\mathcal{E}, g')_0 = (g, g')_0$, $\forall g'$. $\operatorname{div}\mathcal{E} = g$ follows.

(iii) $\mathbf{f} \in \mathbf{H}_0(\operatorname{div}^0, \Omega)$: according to Thm 3.6 p. 48 of [Girault-Raviart'86],
$\exists ! \mathbf{w} \in \mathcal{X}_E^0$ such that $\operatorname{div}\mathbf{w} = 0$, and $\mathbf{curl}\,\mathbf{w} = \mathbf{f}$.
$\mathbf{v} = \mathcal{E} - \mathbf{w} \in \mathcal{X}_E^0$ yields $\|\mathbf{curl}\,(\mathcal{E} - \mathbf{w})\|_0^2 = 0$, so $\mathbf{curl}\,\mathcal{E} = \mathbf{f}$.

(iv) Now, if the electrostatic problem admits two solutions, it is clear that the difference satisfies (P^0) with homogeneous r.h.s., so it is zero; uniqueness follows.

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"Electrostatic" model (3)

- What happens when the domain Ω is not convex? $\mathcal{X}^0_{E,R}$ is *not dense* in $\mathcal{X}^0_E$...
- Remedy: solve the electrostatic problem in a weighted Sobolev space...
- Introduce:
  - The set E of reentrant edges of $\partial\Omega$, and the distance $d_0(\mathbf{x}) = d(\mathbf{x}, E)$.
  - The sets ($\gamma \in [0,1]$)

$$L^2_\gamma(\Omega) := \{g \;:\; g \in L^2_{loc}(\Omega),\ d_0{}^\gamma g \in L^2(\Omega)\},\ \text{with norm } \|g\|_{0,\gamma} = \|d_0{}^\gamma g\|_0 \;;$$

$$\mathcal{X}^0_{E,\gamma} := \{\mathbf{v} \;:\; \mathbf{v} \in \mathbf{H}_0(\mathbf{curl}\,,\Omega),\ \operatorname{div}\mathbf{v} \in L^2_\gamma(\Omega)\}.$$

- Theorem [Costabel-Dauge'02]: $\exists\gamma_0 \in]0,1/2[$, such that
  (i) $\forall\gamma \in]\gamma_0,1]$, the subspace of regular fields $\mathbf{H}^1(\Omega) \cap \mathcal{X}^0_{E,\gamma}$ is *dense* in $\mathcal{X}^0_{E,\gamma}$.
  (ii) $\forall\gamma \in]\gamma_0,1[$, the semi-norm associated to

$$(\cdot,\cdot)_{X^0_\gamma} \;:\; (\mathbf{u},\mathbf{v}) \mapsto (\mathbf{curl}\,\mathbf{u},\mathbf{curl}\,\mathbf{v})_0 + (\operatorname{div}\mathbf{u},\operatorname{div}\mathbf{v})_{0,\gamma}$$

  is a norm in $\mathcal{X}^0_{E,\gamma}$, which is *equivalent* to the full norm.

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Weight Regularization Method

- *Define the AVF* $(\underline{P}^0)$ *in* $\mathcal{X}^0_{E,\gamma}$

$$\begin{cases} \text{Find } \mathcal{E} \in \mathcal{X}^0_{E,\gamma} \text{ such that} \\ (\mathcal{E}, \mathbf{v})_{X^0_\gamma} = (\mathbf{f}, \mathbf{curl}\,\mathbf{v})_0 + (g, \operatorname{div}\mathbf{v})_{0,\gamma}, \quad \forall \mathbf{v} \in \mathcal{X}^0_{E,\gamma}. \end{cases}$$

Theorem: $\exists! \mathcal{E} \in \mathcal{X}^0_{E,\gamma}$ *solution to* $(\underline{P}^0)$.
It is the only solution to the electrostatic model.

- Numerical approximation with the Continuous P_k Lagrange FE is possible:
  - convergence results: $||\mathcal{E} - \mathcal{E}_h||_{X^0_\gamma} \leq C_{\mathbf{f},g}\, C_\varepsilon h^{\gamma-\gamma_0-\varepsilon},\ \forall \varepsilon > 0$ (for $k \geq 2$, cf. [Costabel-Dauge'02]);
  - comparisons and numerical experiments in 2D: cf. [Jamelot'05].

- According to M. Costabel and M. Dauge (private communication):
Similar results are also valid for finding $\mathcal{H}$ in $\mathcal{X}^0_{H,\gamma}$...

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AVF for the time-dependent equations

- Without a Silver-Müller boundary condition ($\Gamma_A = \emptyset$):
  Find $\mathcal{E}(t) \in \mathcal{X}^0_{E,\gamma}$ such that

$$(2) \qquad < \partial^2_{tt}\mathcal{E}, \mathbf{v} > + c^2(\mathcal{E}, \mathbf{v})_{X^0_\gamma} = -\frac{1}{\varepsilon_0}(\partial_t \mathcal{J}, \mathbf{v})_0 + \frac{c^2}{\varepsilon_0}(\rho, \operatorname{div}\mathbf{v})_{0,\gamma}, \ \forall \mathbf{v} \in \mathcal{X}^0_{E,\gamma}.$$

$$\Big(\partial^2_{tt}\mathcal{J} \in L^2(0,T;\mathbf{L}^2(\Omega)), \partial_t\rho \in \mathcal{C}^0(0,T;L^2_\gamma(\Omega)). \Big)$$

- With a Silver-Müller boundary condition ($\Gamma_A \neq \emptyset$):
  - replace $\mathcal{X}^0_{E,\gamma}$ by $\mathcal{X}^A_{E,\gamma} := \{\mathbf{v} \in \mathcal{T}_E \ : \ \operatorname{div}\mathbf{v} \in L^2_\gamma(\Omega)\}$;
  - add the boundary terms of (1) in (2)...
    Find $\mathcal{E}(t) \in \mathcal{X}^A_{E,\gamma}$ such that

$$(3) \left\{ \begin{array}{l} < \partial^2_{tt}\mathcal{E}, \mathbf{v} > + c^2(\mathcal{E}, \mathbf{v})_{X^0_\gamma} + c\frac{d}{dt}(\mathcal{E}_T, \mathbf{v}_T)_{0,\Gamma_A} \\ \qquad = -\frac{1}{\varepsilon_0}(\partial_t\mathcal{J}, \mathbf{v})_0 + \frac{c^2}{\varepsilon_0}(\rho, \operatorname{div}\mathbf{v})_{0,\gamma} + c\frac{d}{dt}(\vec{\mathbf{e}}^\star_T, \mathbf{v}_T)_{0,\Gamma_A}, \ \forall \mathbf{v} \in \mathcal{X}^A_{E,\gamma}. \end{array} \right.$$

Mixed AVF for the time-dependent equations

- Coupling with the Vlasov equation (Particle methods):
  - At the discrete level: $\partial_\tau \rho_h + \mathrm{div}\,_h \mathcal{J}_h \neq 0$.
  - Need of a Lagrange multiplier on $\mathrm{div}\,\mathcal{E}$.

- The mixed AVF (case $\Gamma_A = \emptyset$):
  Find $(\mathcal{E}(t), p(t)) \in \mathcal{X}^0_{E,\gamma} \times L^2_\gamma$ *such that*

(4)
$$\left\{\begin{array}{l} < \partial^2_{tt}\mathcal{E}, \mathbf{v} > + c^2(\mathcal{E}, \mathbf{v})_{X^0_\gamma} + (p, \mathrm{div}\,\mathbf{v})_{0,\gamma} \\ \qquad = -\frac{1}{\varepsilon_0}(\partial_t \mathcal{J}, \mathbf{v})_0 + \frac{c^2}{\varepsilon_0}(\rho, \mathrm{div}\,\mathbf{v})_{0,\gamma}, \ \forall \mathbf{v} \in \mathcal{X}^0_{E,\gamma}\,; \\ (\mathrm{div}\,\mathcal{E}, q)_{0,\gamma} = \frac{1}{\varepsilon_0}(\rho, q)_{0,\gamma}, \ \forall q \in L^2_\gamma. \end{array}\right.$$

$$\left(\partial^2_{tt}\mathcal{J} \in L^2(0,T;\mathbf{L}^2(\Omega)),\, \partial^2_{tt}\rho \in L^2(0,T;L^2_\gamma(\Omega)). \right)$$

- The mixed AVF (case $\Gamma_A \neq \emptyset$):
  replace $\mathcal{X}^0_{E,\gamma}$ by $\mathcal{X}^A_{E,\gamma}$ and add the boundary terms of (1) in (4)...

Discretization

- In time: leap-frog scheme

$$\partial_{tt}^2 u(t_n) \equiv \frac{u(t_{n+1}) - 2u(t_n) + u(t_{n-1})}{(\Delta t)^2}.$$

- In space, a continuous Galerkin Method:
  - P_k Lagrange FE, or $P_{k+1} - P_k$ Taylor-Hood FE ;
  - $P_2 - iso - P_1$ Taylor-Hood FE is possible ([Assous et al'93] in the convex case).
- Overall, an explicit discretization scheme:

$$\begin{cases} \left(\mathbb{M}_\Omega + \frac{c\Delta t}{2}\mathbb{M}_{A,\|}\right)\vec{\mathsf{E}}^{n+1} + (\Delta t)^2\mathbb{C}^T\vec{\mathsf{p}}^{n+1} = \vec{\mathsf{f}}^{n+1/2} \\ \mathbb{C}\vec{\mathsf{E}}^{n+1} = \vec{\mathsf{g}}^{n+1}. \end{cases}$$

  Under a CFL: $c\,\Delta t \leq C_k\ \min_l h_l$.

- Mass lumping is possible [Cohen'02]: $\widetilde{P}_1$ or $\widetilde{P}_2$ FE $\rightsquigarrow$ fully explicit scheme.

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Discretization (2)

- Is the Lagrange multiplier (MAVF) mandatory?
  - Not really, except for Vlasov-Maxwell?! (cf. [Garcia'02])
  - Computed once every 10 (or more) time-steps...
  - Use the Preconditioned CG method to compute $\vec{p}^{n+1}$ (cf. [Jamelot'05].)
- convergence result for the implicit scheme (see [Jr-Labrunie'06]):

$$\max_n \left(||\partial_t \mathcal{E}(t_n) - \partial_\tau \mathcal{E}_h^n||_0^2 + ||\mathcal{E}(t_n) - \mathcal{E}_h^n||_{X_\gamma^0}^2 \right) \\ \leq C_\varepsilon \left((\Delta t)^2 + h^{2(\gamma-\gamma_0-\varepsilon)} + (\Delta t)^2 h^{2(\gamma-\gamma_0-1-\varepsilon)} \right), \ \forall \varepsilon > 0.$$

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Numerical examples

- Computation of the electromagnetic field in a closed convex cavity (cf. [Heintzé'92]):
  - no source terms ($(\mathcal{J}, \rho) = (0, 0)$) ;
  - no artificial boundary ($\Gamma_A = \emptyset$) ;
  - P_1, $\widetilde{P}_1$ or P_2 FE on 25K tetrahedra.

- Computation of the electromagnetic field around a non-convex body:
  - generated by a current ($\mathcal{J} \neq 0$, $\rho \neq 0$) ;
  - absorbing boundary condition on Γ_A ;
  - P_1 or $\widetilde{P}_1$ FE on 684K tetrahedra.

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Computation of the EM field in a cavity

- Solving Maxwell equations in a unit cube (no source terms, non-zero I. C.):
  Find $\mathcal{E}(t) \in \mathcal{X}^0_{\mathcal{E}}$ *and* $\mathcal{H}(t) \in \mathcal{X}^0_{\mathcal{H}}$ *such that*

$$
\begin{aligned}
< \partial^2_{tt}\mathcal{E}, \mathbf{v} > + c^2\,(\mathcal{E}, \mathbf{v})_{X^0} = 0,\ \forall \mathbf{v} \in \mathcal{X}^0_{\mathcal{E}} \\
< \partial^2_{tt}\mathcal{H}, \mathbf{v} > + c^2\,(\mathcal{H}, \mathbf{v})_{X^0} = 0,\ \forall \mathbf{v} \in \mathcal{X}^0_{\mathcal{H}}.
\end{aligned}
$$

- Exact solution:

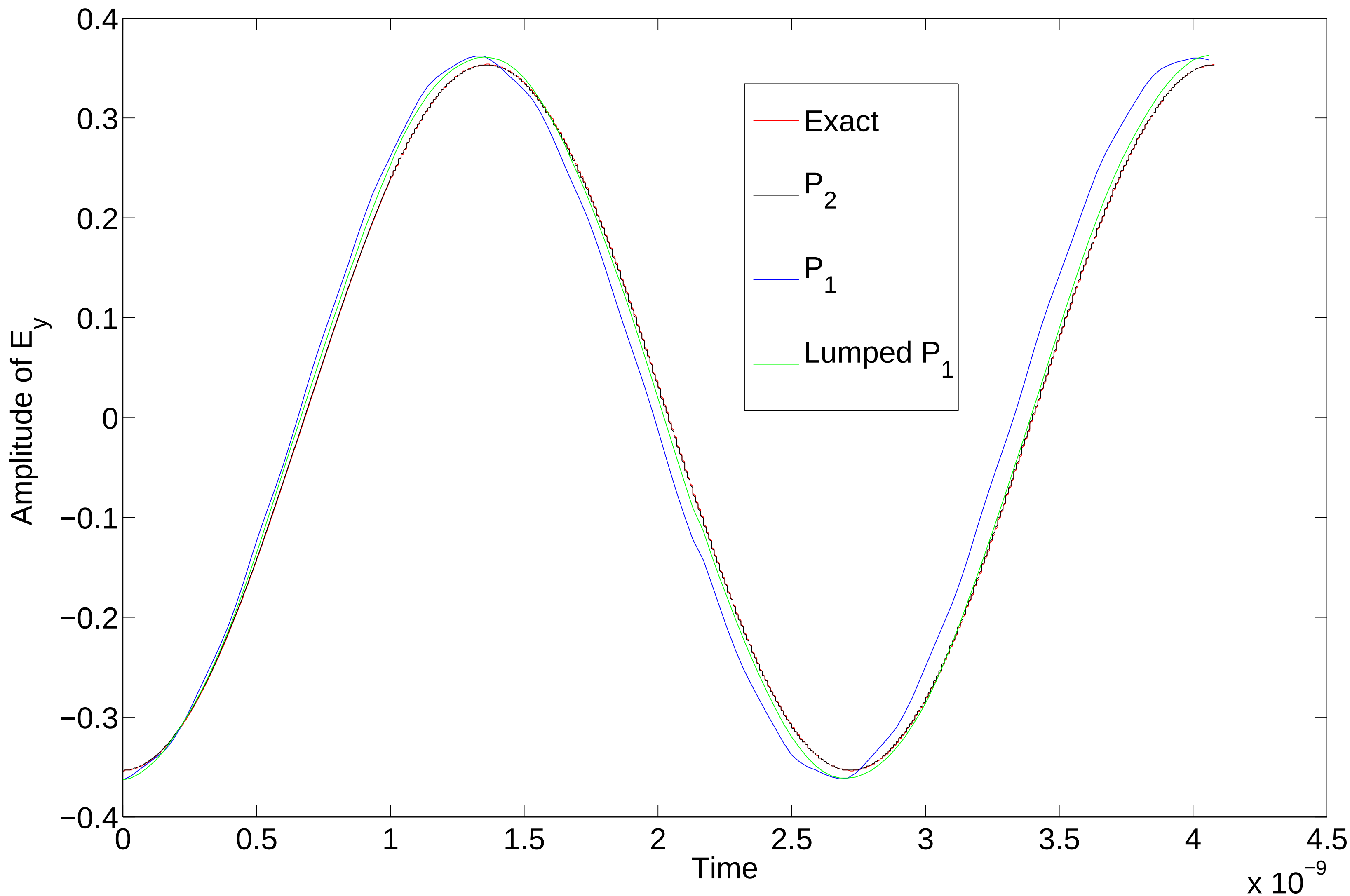
$$
\mathcal{E}(t) = \cos(\omega t) \begin{pmatrix} \cos(\pi x)\ \sin(\pi y)\ \sin(-2\pi z) \\ \sin(\pi x)\ \cos(\pi y)\ \sin(-2\pi z) \\ \sin(\pi x)\ \sin(\pi y)\ \cos(-2\pi z) \end{pmatrix},
$$

$$
\mathcal{H}(t) = \frac{3\pi}{\mu_0 \omega} \sin(\omega t) \begin{pmatrix} -\sin(\pi x)\ \cos(\pi y)\ \cos(-2\pi z) \\ \cos(\pi x)\ \sin(\pi y)\ \cos(-2\pi z) \\ 0 \end{pmatrix}.
$$

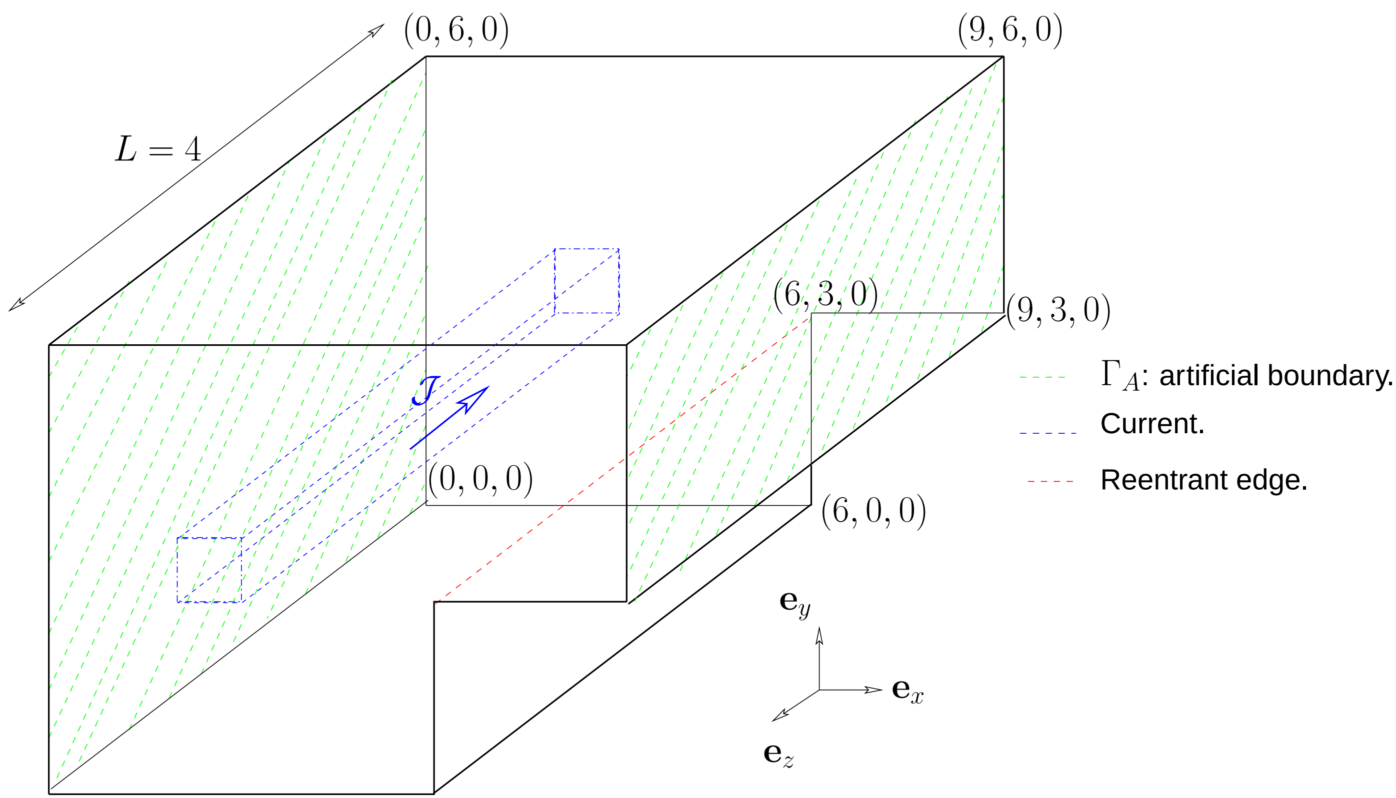
- $c \approx 3.0 \times 10^8\,\mathrm{m.s}^{-1}$, $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{H.m}^{-1}$, $\omega \approx 2.3 \times 10^9\,\mathrm{Hz}$.
- 10 discretization nodes per wave length $\rightsquigarrow$ 25K tetrahedra.

P_1, $\widetilde{P_1}$ and P_2 FE

- $\mathcal{E}_y$ relative amplitude at point $(0.19, 0.12, 0.12)$.



Some results without Lagrange multiplier, $\widetilde{P}_1$ FE



- $c \approx 3.0\,10^8$ m.s^{-1}, $\omega \approx 2.5\,10^9$ Hz.
- $\mathcal{J} = 10^{-5}\,\omega\,\sin\left(\frac{\pi\,z}{L}\right)\,\cos(\omega\,t)\mathbf{e}_3$, $\rho = 10^{-5}\,\frac{\pi}{L}\,\cos\left(\frac{\pi\,z}{L}\right)\,\sin(\omega\,t)\,;\,\mathcal{E}_0 = \mathcal{H}_0 = 0.$
- No incoming wave: $\mathbf{e}^\star = 0$.
- 684K tetrahedra.

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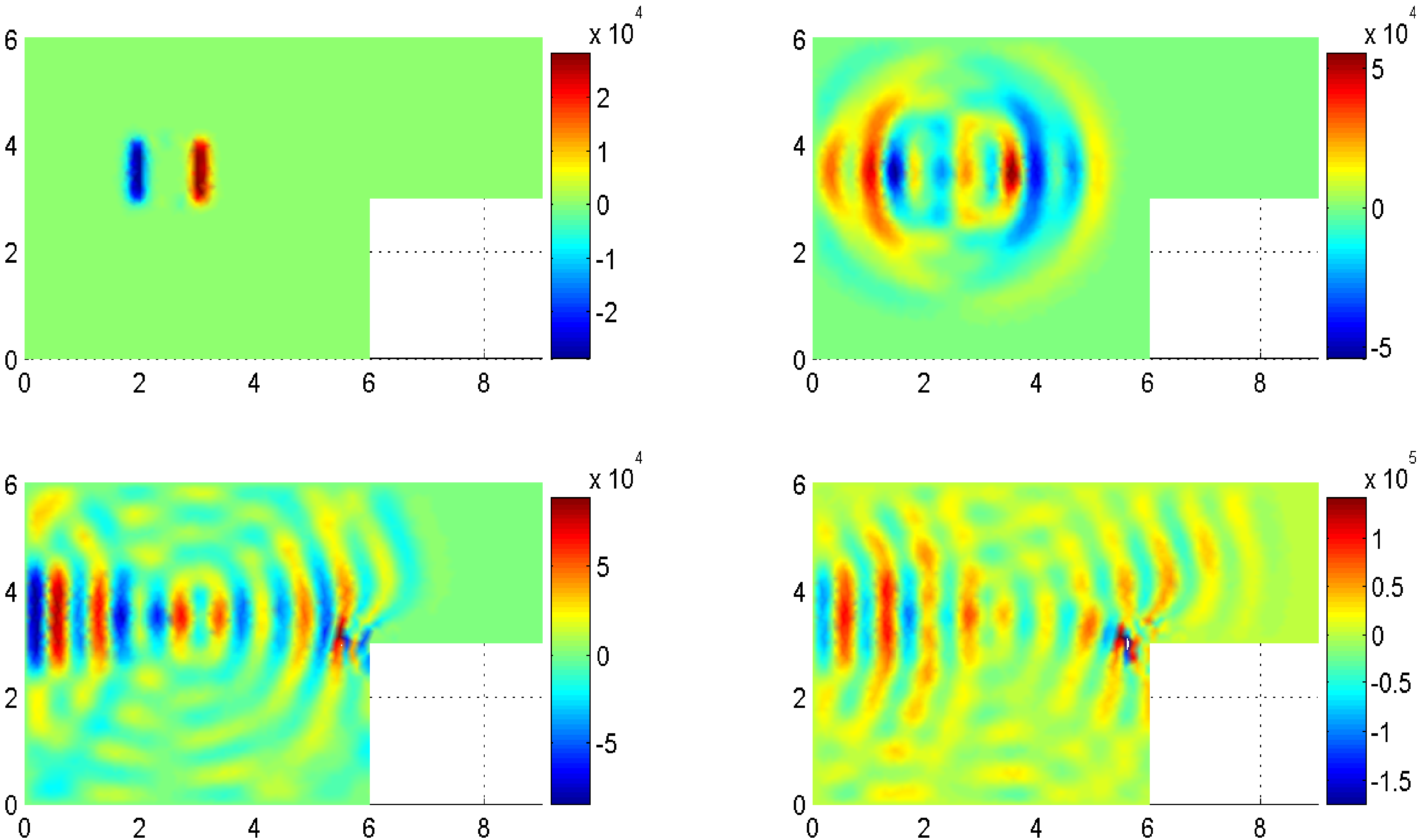
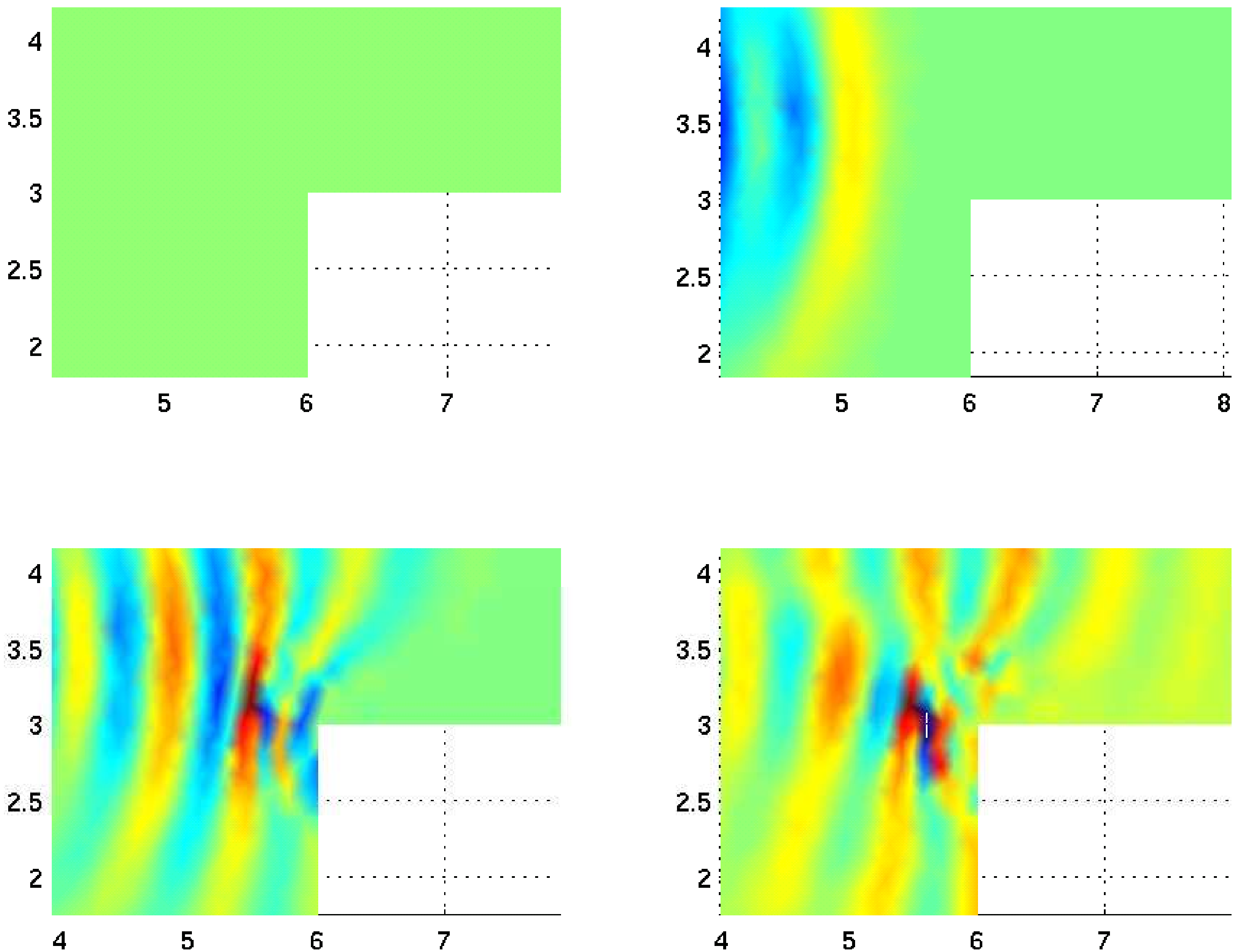
$\mathcal{E}_{h,x}$ in plane $z = 2.5$: space evolution

- $T_1 = 1$ns, $T_2 = 8$ns, $T_3 = 15$ns, $T_4 = 20$ns.



$\mathcal{E}_{h,x}$ in plane $z = 2.5$: zooming in...

- $T_1 = 1\text{ns}$, $T_2 = 8\text{ns}$, $T_3 = 15\text{ns}$, $T_4 = 20\text{ns}$.



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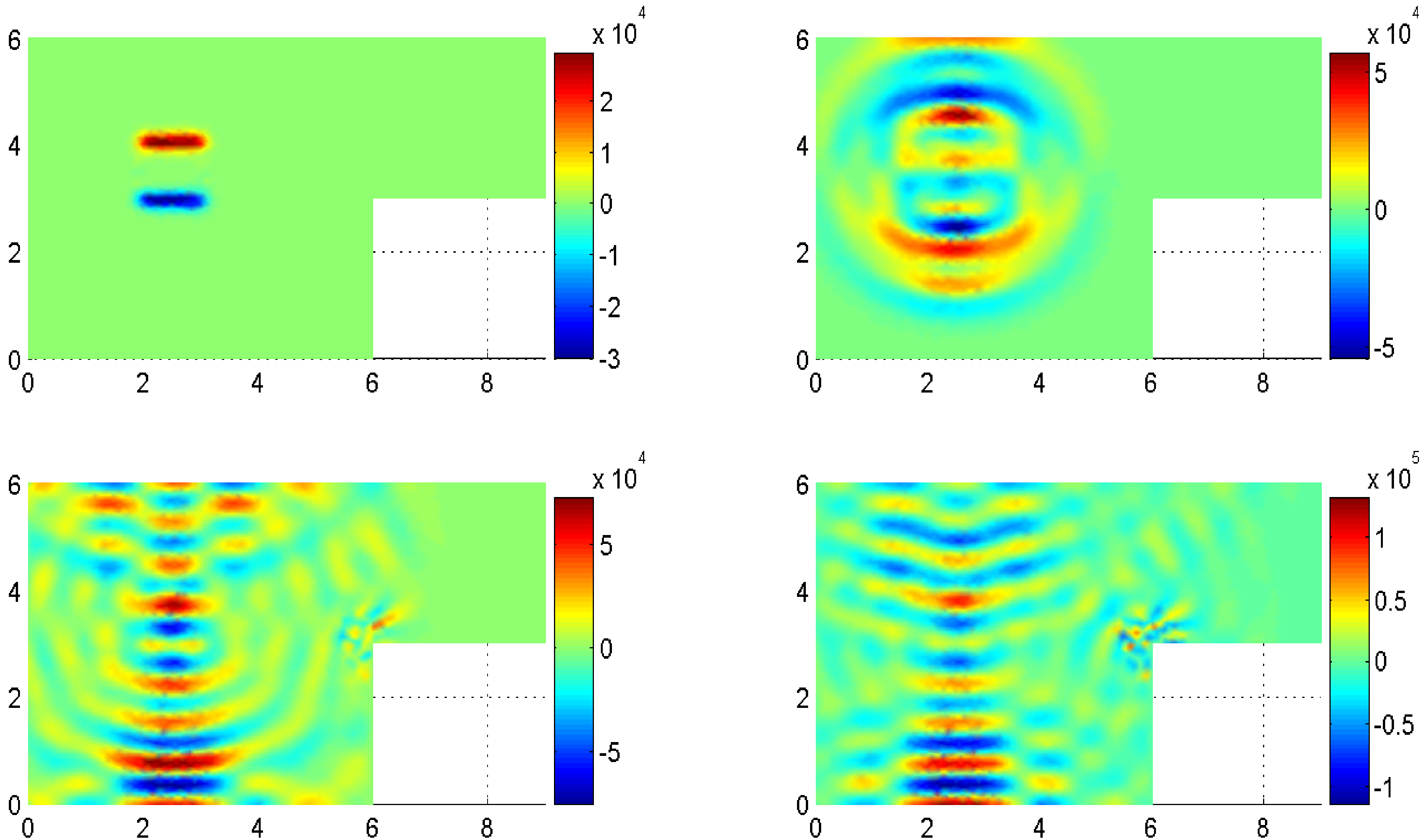
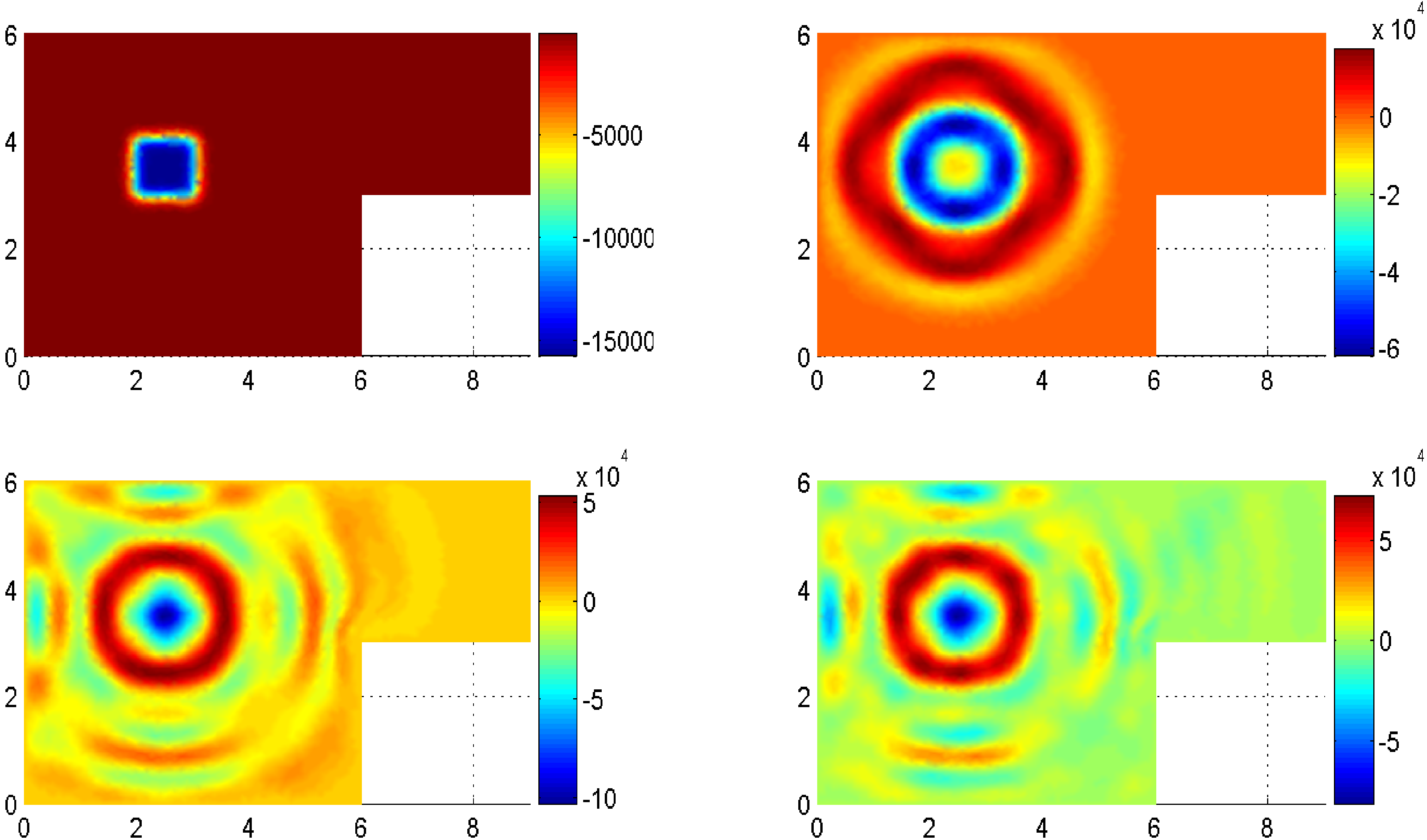
$\mathcal{E}_{h,y}$ in plane $z = 2.5$: space evolution

- $T_1 = 1\text{ns}$, $T_2 = 8\text{ns}$, $T_3 = 15\text{ns}$, $T_4 = 20\text{ns}$.



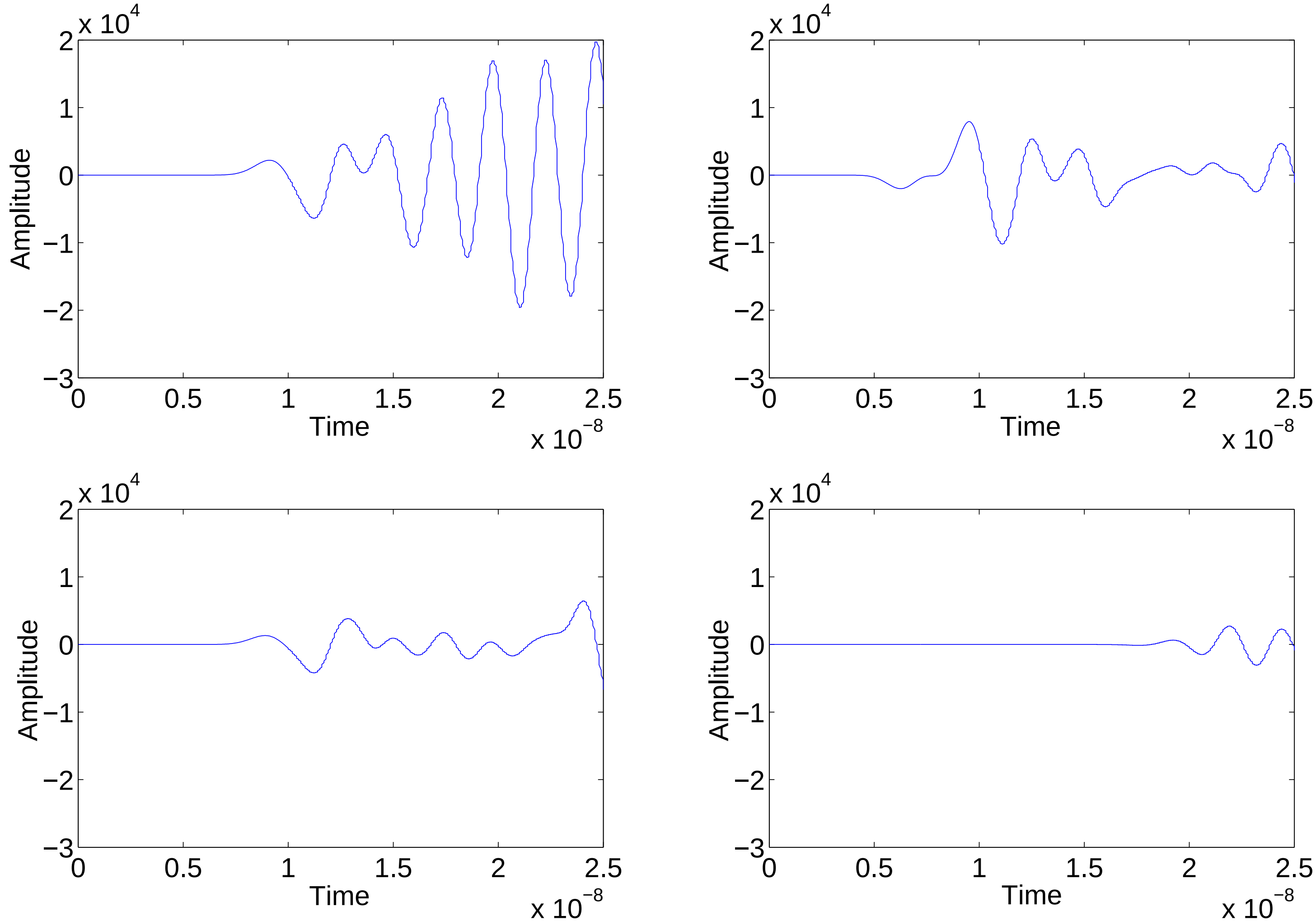
$\mathcal{E}_{h,z}$ in plane $z = 2.5$: space evolution

- $T_1 = 1$ns, $T_2 = 8$ns, $T_3 = 15$ns, $T_4 = 20$ns.



$\mathcal{E}_{h,x}$: time evolution

- $M_1 = (1, 1, 2)$, $M_2 = (1, 5, 2)$, $M_3 = (5.5, 2.5, 2)$, $M_4 = (8, 5.5, 2)$.



Conclusion/Perspectives

- One can solve numerically Maxwell equations with continuous Galerkin methods! (cf. http://www.ensta.fr/~jamelot/)
- The numerical implementation is not very costly, and one can use mass lumping...
- To achieve better precision:
  - increase k, the order of the FE ;
  - use PMLs to close the domain.

- The Mixed AVF can be useful to solve:
  - the coupled Vlasov-Maxwell system of equations (ongoing project with F. Assous) ;
  - eigenvalue problems [Buffa-Jr-Jamelot'06].

- Alternate methods:
  - 2D, 3D: the Natural Boundary Condition Method ([Jr'05], [Jamelot'05]).
  - 2D, 2D1/2: the Singular Complement Method ([Garcia'02], [Labrunie et al'0...], [Jamelot'05]).

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