PhD proposal: Nonlocal models for interface problems between dielectrics and metamaterials in electromagnetism

The aim of the thesis is to develop methods to solve problems in electromagnetism for configurations with both classical materials (dielectrics) and metamaterials. This association of dielectrics and such metamaterials has very exciting potential applications in physics such as plasmonic waveguides, perfect lenses, photonic traps, subwavelength cavities, etc.. The idea is to model and study them from a mathematical point of view.
In particular, this type of configuration involves an interface between those materials and, at this interface, the electric permittivity and/or the magnetic permeability change sign. Because of the sign-change of the coefficients, the usual coercivity properties of the corresponding variational formulation are lost. The T-coercivity approach has been designed to address this difficulty [1]. However, when the interface has corners and/or edges, the formulation can be ill-posed [2] in some configurations, depending on the values of the coefficients at the interface. Considering the problem with a scalar unknown (ie. one of the components of the electromagnetic field), this means that one may not be able to recover "finite-energy" solutions, where the energy is measured according to the classical H^1 norm. A possible way to recover well-posedness is then to consider a different framework, which consists in choosing, in the variational formulation, enlarged function spaces, bigger than the H^1 space, that include (very) singular solutions [3]. Numerically, this approach raises new challenges which have been partially addressed only recently [4].
In this thesis, we would like to investigate a novel approach, which consists in reformulating the interface problem within a nonlocal setting. In short, to replace the (local) differential operators by nonlocal counterparts [5]. The preliminary results are very encouraging. In [6], numerical evidence is shown that suggests that the discretization of the nonlocal interface problem converges in configurations where the local variational formulation is ill-posed. See Figure 1 for a numerically stable nonlocal result, whereas the classical model is known to be ill-posed, including a square metamaterial (bottom right) and a dielectric elsewhere.

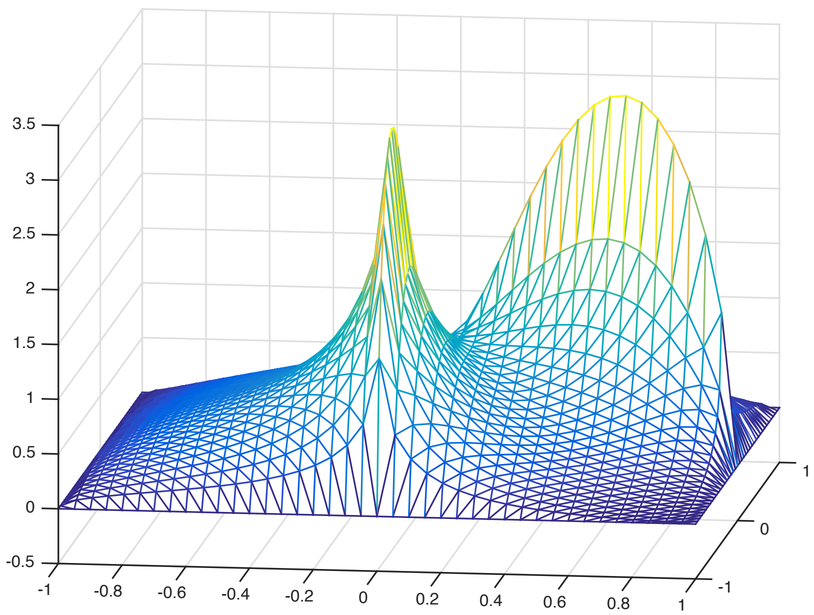

Fig. 1

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Nonlocal models for interface problems between dielectrics and metals or metamaterials,
Proceedings of the International Congress Metamaterials Marseille, Aug. 2017

Candidate profile
Master in Mathematics or Applied Mathematics. Solid background in functional analysis ; strong taste for mathematical modelling ; basic programming skills.

PhD advisor
Patrick Ciarlet
ENSTA ParisTech
Laboratoire POEMS
Palaiseau, FRANCE
email: patrick.ciarlet@ensta-paristech.fr