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Conference Paper · July 2016

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DIVERGENCE MODES OF A CLUSTER OF SIMPLY SUPPORTED ARRAYS IN AXIAL FLOW

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ABSTRACT

This paper is a numerical study of the static divergence patterns that could be observed upon a cluster of flexible cylinders in axial water flow. Previous experimental work has shown that beyond a critical flow velocity, characteristic deformation patterns can indeed be observed for square cylinder arrays in axial flow. This paper attempts to rewrite the physical problem in mathematical terms in which the fluid and cylinder coupling forces are modeled using influence coefficients. These coefficients are calculated and compared using two methods: simple potential flow and CFD including more complex turbulence modeling. The full static divergence problem is then numerically solved for an example system of a 10x10 cylinder arrays. Finally, the results for the divergence modes are visually presented and the intervening physical couplings are commented.

NOMENCLATURE

$F_{i,j}^{x}$: horizontal (along x) fluid force acting on a cylinder displaced along the same (x) direction. [N]

$F_{i,j+1}^{x}$: horizontal force acting on a horizontally adjacent cylinder (same row, incremented column) to a central cylinder displaced along the horizontal direction. [N]

$F_{i+1,j}^{x}$: horizontal force acting on a vertically adjacent (same column, incremented row) cylinder to a central cylinder displaced along the horizontal direction. [N]

$F_{i+1,j+1}^{x}$: horizontal force acting on a diagonally adjacent cylinder to a central cylinder displaced along the horizontal direction. [N]

$F_{i+1,j+1}^{y}$: vertical (y) force acting on a diagonally adjacent cylinder to a central cylinder displaced along the horizontal direction. [N]

C_{self} : self force-to-displacement influence coefficient. [*dimensionless*]

$C_{//,\perp,diag}$: inter cylinder force-to-displacement influence coefficients. [*dimensionless*]

ρ : fluid density [kg.m^{-3}]

U : fluid velocity [m.s^{-1}]

D : cylinder diameter [m]

X_{ij} : deformation amplitude of cylinder (i,j)

INTRODUCTION

Slender structures submitted to axial flows are known to undergo flutter-like instabilities when the velocity reaches a sufficient value. In the case of structures fixed or clamped at both ends, a static instability known as divergence occurs first, generating large deformations and breaking the axial symmetry of the array. Païdoussis and co-authors have observed and studied this phenomenon in the 80s with arrangements of three and four silicon cylinders in a water tunnel [1]. The present numerical study aims at predicting the global pattern that would be obtained in a similar experiment with a higher number of cylinders, so that boundary effects would not dominate the instability. A square array of 10 by 10 straight cylinders is considered, with diameters of the order of 1 cm, a pitch to diameter ratio equal to 1.4, a reduced length L/D equal to 25, and an incipient fluid velocity equal to 1 m/s, so that the Reynolds number would be of the order of 10000 for water in ambient conditions.

MODELING FRAMEWORK

The array behavior is modeled with the help of simplifying assumptions: first, the cylinders deformations are reduced to sine-like first mode shapes only, and second, fluid-structure interactions are described with the help of influence coefficients in the manner of Tanaka [2] by assuming that a small displacement of a cylinder generates forces on the eight surrounding cylinders only. Denoting Xij the displacement amplitude of the i-th row / j-th column cylinder in the x direction, the modal forces generated by this displacement are with self-evident notations:

$$F_{ij}^{x} = C_{self}\, \rho U^2 D\, X_{ij}$$
$$F_{ij+1}^{x} = C_{//}\, \rho U^2 D\, X_{ij}$$
$$F_{i+1j}^{x} = C_{\perp}\, \rho U^2 D\, X_{ij}$$
$$F_{i+1\,j+1}^{x} = C_{diag\_1}\, \rho U^2 D\, X_{ij}$$
$$F_{i+1\,j+1}^{y} = C_{diag\_2}\, \rho U^2 D\, X_{ij}$$

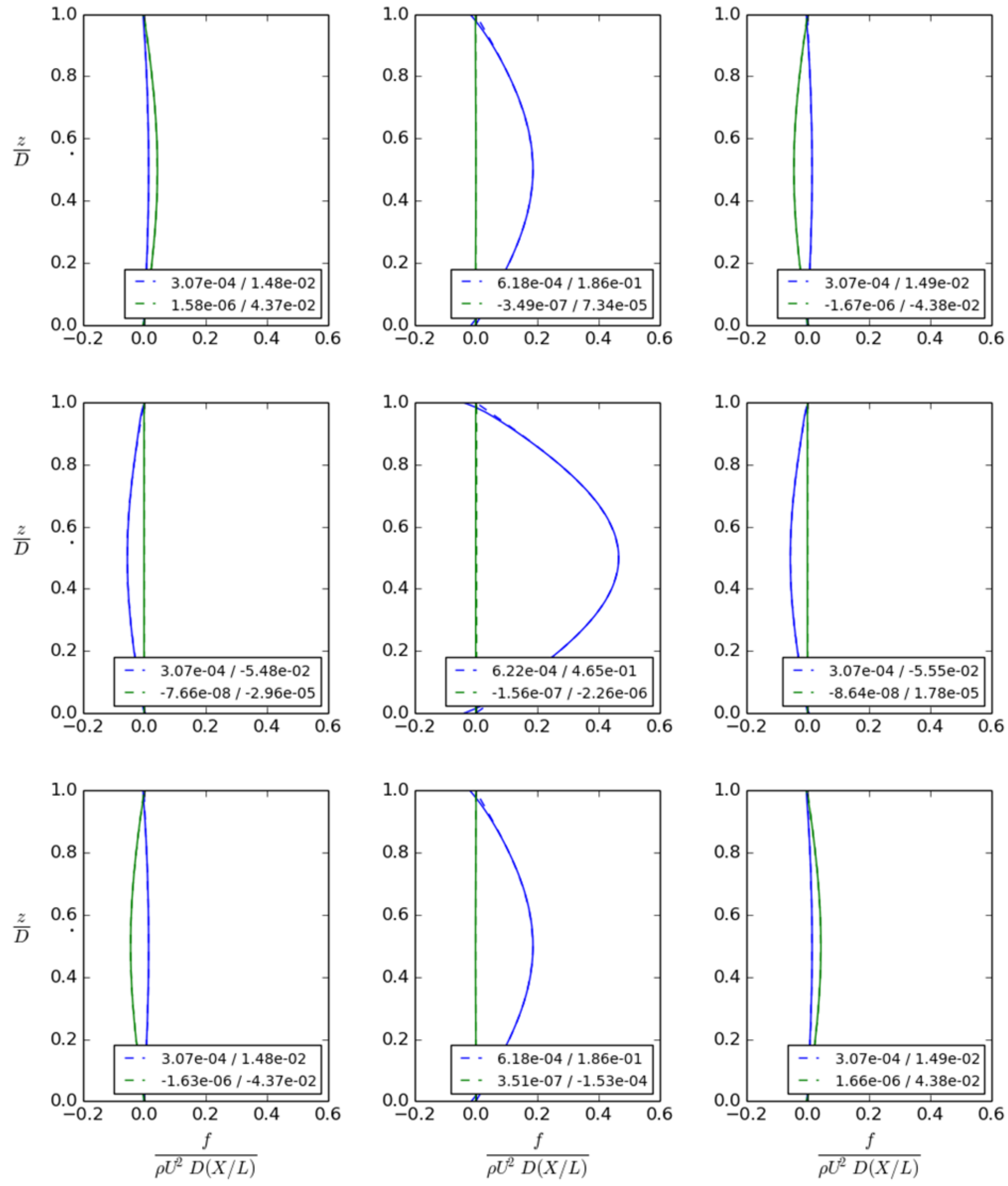

Figure 1: Dimensionless fluid force densities in the x (blue) and y (green) directions, for a deformation of the central cylinder in the *x* direction. Legend: least-square fit coefficients for sine and cosine, respectively. Results obtained by a potential calculation with code_aster [3]

the other terms can be obtained with the help of symmetry considerations.

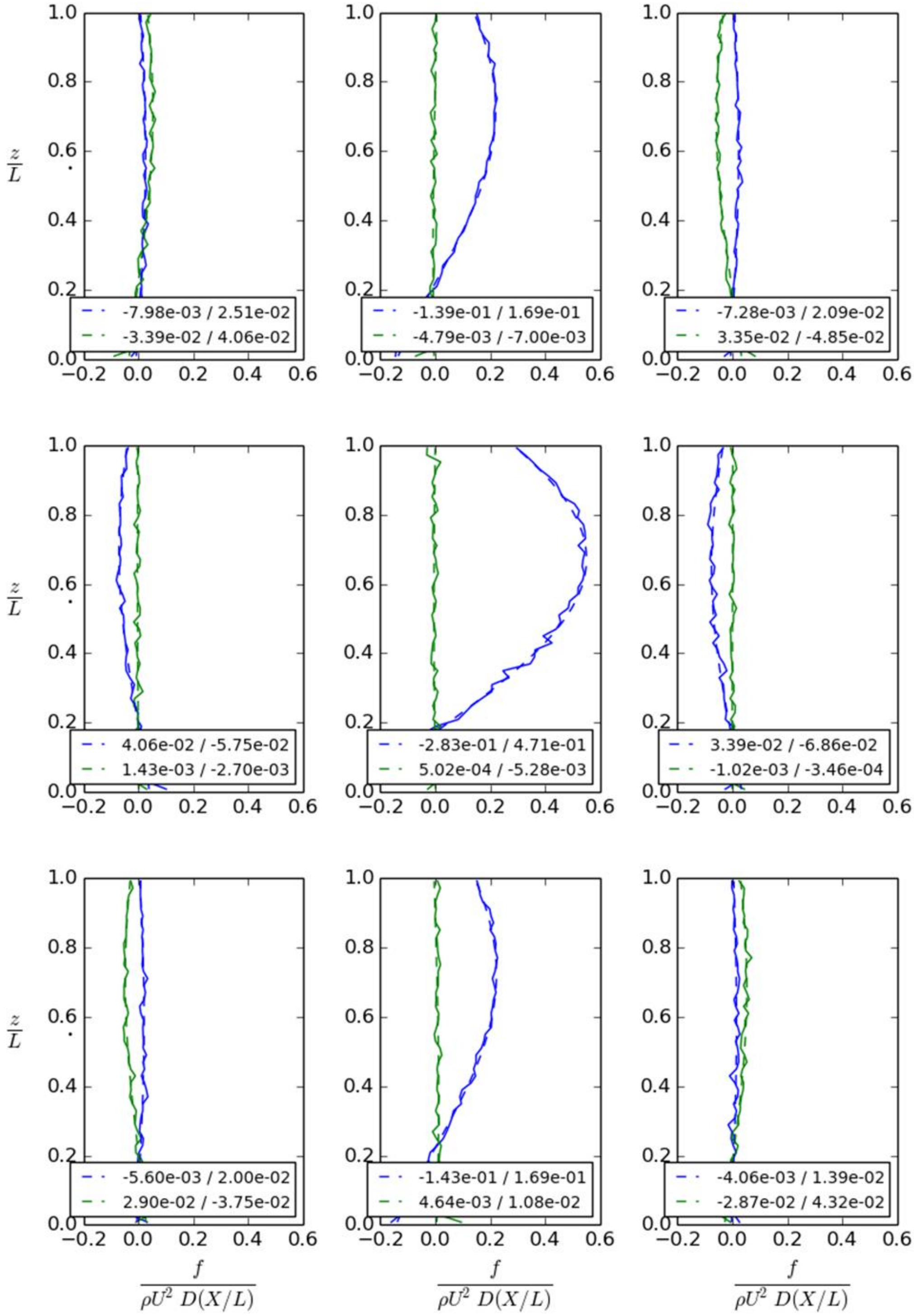

Figure 2: Dimensionless fluid force densities in the x (blue) and y (green) directions, for a deformation of the central cylinder in the *x* direction. Legend: least-square fit coefficients for sine and cosine, respectively. Results obtained by averaging a k-omega SST calculation with code_saturne [4]

INFLUENCE COEFFICIENTS

Estimations of the previous influence coefficients are provided by FEM and CFD simulations: a first series of calculations is performed using a potential flow modeling, and a second with a k-omega SST turbulent modeling, as illustrated in Figs 1 and 2. A cylinder with a deflection X/D equal to 0.005 in the x-direction is arranged in the middle of a square array of straight cylinders, and ideal walls without slip surround this array at a distance equal to half a gap. The velocity flux is prescribed upstream, and the pressure or the potential is prescribed downstream throughout a couple of 'buffer' areas which tranquilize the flow.

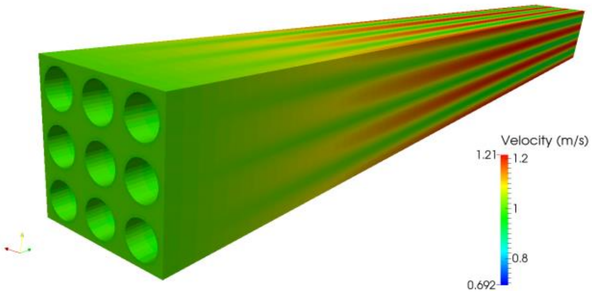

Figure 3: Upstream view of the velocity field obtained by a k-omega SST calculation. A boundary layer is generated after several diameters.

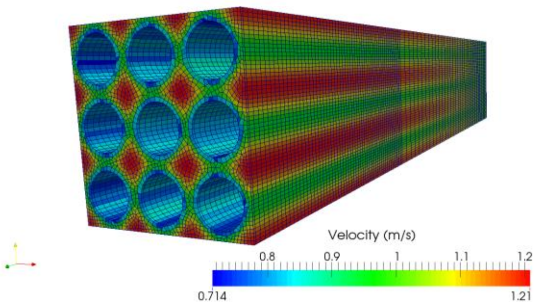

Figure 4: Cross section of the fluid volume at mid-length of the cylinders. Velocity field obtained by a k-omega SST calculation.

The general trend observed in both series of calculations is that two cylinders are attracted towards each other when the gap is reduced, and repulsed when it is increased. A less intuitive result is that the highest coupling occurs between two cylinders in the direction perpendicular to the displacement. As can be observed, the fluid force is much higher in the case of the turbulent calculation, and it exhibits a spacewise 'phase shift'. Careful experiments would be required to discuss these results, yet this discrepancy is understandable in view of the dissipative lift force exerted upon cylinders in quasi-axial flow [1, 5], which cannot be derived from a potential approach, but can be predicted by standard RANS simulations. In the first case, the conservative fluid force term is approximately proportional to the cylinder deformation, whereas in the second case, the dissipative lift force term is proportional to the cylinder angle. The combination of these two terms brings out a sine-like distribution of the fluid force, with an apparent phase shift. Furthermore, as shown in Fig. 3, a boundary layer is generated along the cylinders so that the fluid velocity is higher than 1 m/s in the inner channels. This velocity variation tends to increase the conservative term of the fluid force compared to the purely potential case.

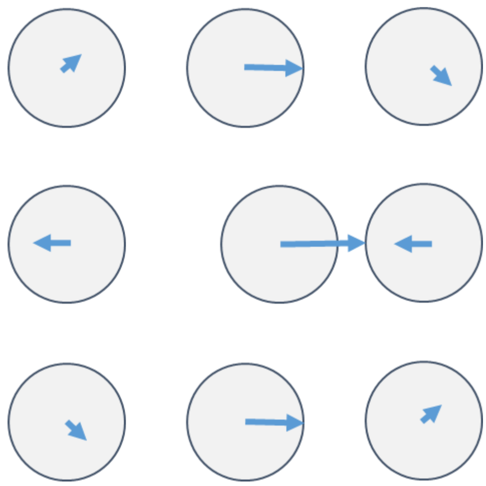

Figure 5: Force pattern due to a lateral displacement of the central cylinder

DIVERGENCE AS AN EIGENVALUE PROBLEM

Divergence occurs when the negative fluid stiffness becomes equal to the structure's one. In mathematical terms, this marginal equilibrium condition can be written as the existence of a non-vanishing solution to the equation:

$$a\frac{EI}{L^3}\begin{pmatrix}X_{ij}\\Y_{ij}\end{pmatrix} = \rho U^2[\boldsymbol{C}]\begin{pmatrix}X_{ij}\\Y_{ij}\end{pmatrix}$$

where the terms between brackets in the above equation are column vectors, standing for all the kinematic degrees of freedom of the cylinders, where EI/L^3 is the bending stiffness of a cylinder, α a modal projection coefficient, and where **C** is the dimensionless fluid stiffness matrix built up with the coefficients of the previous section. Non vanishing solutions of the above system exist when $\alpha EI/\rho U^2 L^3$ is equal to an eigenvalue of **C**. Hence, the first critical velocity U_{crit} can be determined by calculating the eigenvalues of **C** and picking up the highest one λ_{max} ; U_{crit} is then equal to the square root of $\alpha EI\ /\rho\ \lambda_{max}\ L^3$. More details about this approach are available in [6].

APPLICATION TO A SQUARE ARRAY OF SLENDER CYLINDERS

The perspective of the current paper consists in designing a future test rig dedicated to the observation of divergent modes with a significantly high number of flexible structures. For this purpose, attention should be paid about the critical velocity and also about the deformation patterns associated with divergence. The procedure described previously can be applied to the calculation of these patterns; it is just a matter of degrees of freedom ordering, namely the directions of deformation, the mode shape order, and the x/y position of each cylinder of the array. With these considerations carefully addressed, the whole fluid stiffness matrix **C** can be assembled in a straightforward manner, which will not be detailed hereafter.

Potential influence coefficients are introduced for determining **C**, and the eigenvalue problem is solved using the Arnoldi power iteration approach. Results are plotted in Figs. 6 to 10.

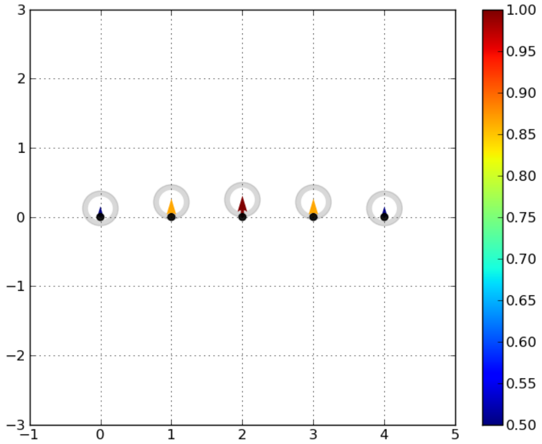

Figure 6: Lateral divergence mode in a single row of cylinders

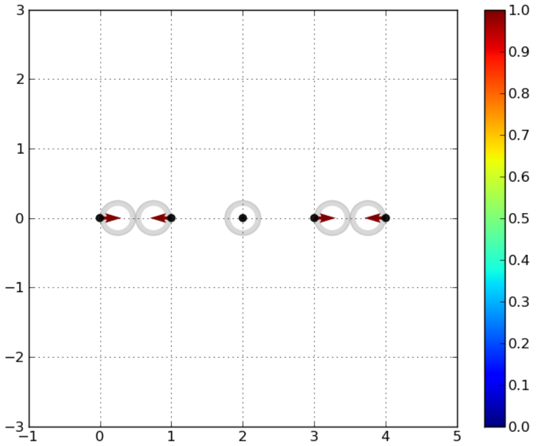

Figure 7: Cylinder pairs pattern generated by an in-line divergent mode in a single row of cylinders

The simple case of a one row by five columns array illustrates the driving mechanisms of divergent modes: in Fig. 6, the lateral displacement of the central cylinder induces a similar displacement of its neighbors, whereas in Fig. 7, the in-line displacement of one cylinder attracts its closest neighbor, and repulses its opposite, a series of attracted pairs of cylinders being generated step by step.

A similar trend can be observed at a larger scale in a 10 x 10 array, where cylinder pairs are generated in one direction, and global displacements in the other direction, as shown in Fig. 8.

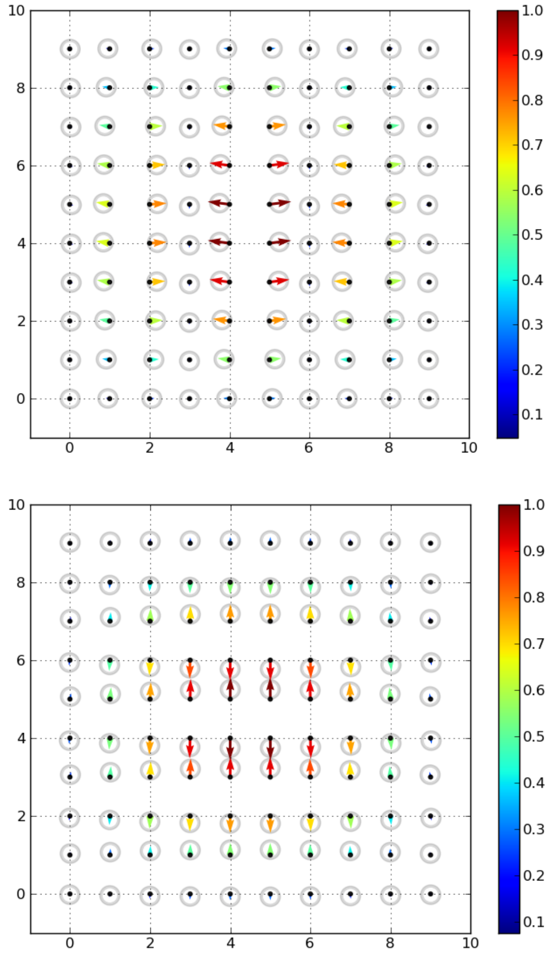

Figure 8: In-line cylinder pairs coupled to lateral coupling in a 10 by 10 array of cylinders

More complex patterns can also appear as shown in the graphs of figures 9 and 10.

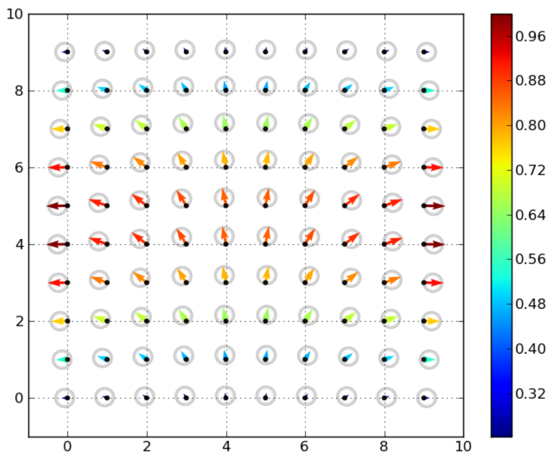

Figure 9: a two dimensional divergence pattern in a 10 by 10 array of cylinders

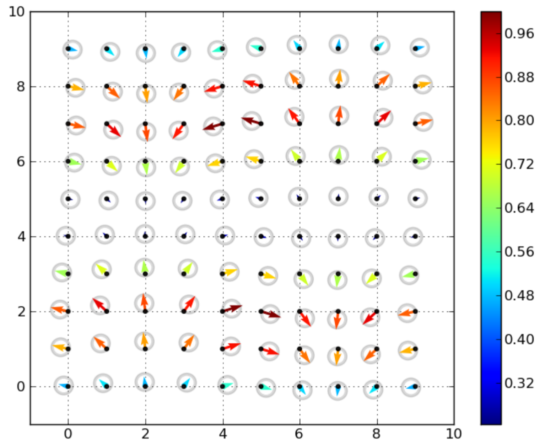

Figure 10: a higher order two dimensional divergence pattern in a 10 by 10 array of cylinders

CONCLUSION AND PERSPECTIVES

The preliminary results described in this paper will be put to trial with a test rig currently under construction. Till this moment, it would be preposterous to extensively investigate the influence of numerical parameters like the turbulence model, the number and arrangement of the cylinders, the confinement, or the length-to-diameter ratio.

Therefore, the only perspective that can reasonably be proposed by now is to experimentally test the influence coefficients of a reduced set of cylinders, and afterwards observe the divergence patterns for an increased number of cylinders with carefully controlled mechanical and hydraulic boundary conditions. The authors expect that original collective deformations of the cylinder array will be observed.

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