ECOLE POLYTECHNIQUE
PROMOTION X00
FLISS Sonia

RAPPORT DE STAGE D'OPTION SCIENTIFIQUE

Numerical elastodynamic modeling of earthquake rupture through branched and offset fault systems

Option:	MECANIQUE
Code de l'option:	MEC 592
Organisme d'accueil:	Division of Engineering and Applied Science Department of Earth and Planetary Sciences Harvard University 29 Oxford Street Cambridge MA 02138 USA
Directeur de l'option:	Rachel-Marie Pradeilles Duval
Directeur de stage:	James R. RICE
Dates du stage:	April 14th-August 15th 2003

Abstract

The rupture zones of major earthquakes often involve geometric complexities including branches and step-overs. These geometric characteristics given by the path at the surface of the Earth can help to know the rupture directivity of past earthquakes which is very important for seismic risk assessments for future events. Indeed, some studies assume that all branches are through acute angles in the direction of rupture propagation. In some observed cases rupture paths seem to branch through an obtuse angle, as if to propagate "backwards". However, stress fields never allow obtuse branching angle.

In our study, we show with numerical results that the large-angle branching is very likely to occur by a jumping mechanism. We use the Boundary Integral Equation Method which is believed to be the most suitable approach to dynamic analysis of non-planar crack, to study the eventual rupture jumping between more and more complex configurations of faults. These first analyzes enable us to improve the model to finally apply it to the 1992 Landers earthquake. This study shows that there is no simple correlation between faults' geometry and rupture directivity which propagates along them.

Le système de failles impliquées lors des tremblements de terre de grande magnitude présente souvent, à la surface de la Terre, une géométrie compliquée composée de jonctions et d'écarts. Ces caractéristiques géométriques peuvent donner une indication sur la directivité de la rupture d'anciens tremblements de terres, information importante pour la prévision des risques de séismes. En effet, des études supposent que toutes les branches se constituent suivant un angle aigu par rapport a la direction de la propagation de la rupture. Dans certains cas, les branches semblent former des angles obtus avec cette direction, comme si la rupture pouvait revenir en arrière. Cependant, les champs de tension ne permettent pas à la rupture de se propager le long d'une telle branche.

Dans notre étude, nous montrons numériquement que ce genre de propagation vers "l'arrière" à des grandes chances de s'effectuer via un saut de rupture. Nous utilisons la méthode des équations intégrales de frontières qui est la méthode numérique la plus pratique pour modéliser des fractures non planes, pour étudier l'éventuel saut de rupture entre des failles dont la geométrie deviendra de plus en plus complexe. Ces premières analyses nous permettent par la suite d'améliorer notre modèle et de l'appliquer à certaines failles impliquées dans le tremblement de terre de 1992 de

Landers. Cette étude nous permettra ainsi de montrer qu'il n'y a pas de corrélations simples entre l'empreinte des failles à la surface et la directivité de la rupture se propageant le long d'elles.

Contents

Introduction

Knowledge of the rupture directivity of past earthquakes from surface fault expressions, if possible, would help in seismic risk assessments for future events. In a recent paper, Nakata et al. [17] propose to relate the observed surface branching of fault system with the directivity of dynamic ruptures along them. Their work assumes that all branches are through acute angles in the direction of rupture propagation, like illustrated in Figure 1.

Figure 1: Typical branching through acute angle

However, in some cases rupture paths seem to branch through an obtuse angle, as if to propagate "backwards", as observed in Dmowska et al. [6]. The authors have collected four cases of such backward branching. The one we are going to focus on occurs during 1992 Landers earthquake (Figure 2). When crossing from the Johnson Valley to the Homestead Valley fault via the Kickapoo fault, the rupture from the Kickapoo segment progressed not only forward onto the northern stretch of the Homestead Valley fault, but also backward, i.e., SSE along the Homestead Valley fault ([25]), (Figure 2)(the slip is observed right lateral [25]).

Figure 2: Fault map from Sowers et al., [25]

This example of such strongly obtuse branch suggests that there may be no simple correlation between fault geometry and rupture directivity. A problem to be understood is that of whether these obtuse branches actually involve a rupture path which directly turned through the obtuse angle(while continuying also on the main fault), like illustrated in Figure 3(a), or rather involved a step over of the rupture by nucleation on a neighboring fault. Stress fields as plotted by Poliakov et al. [19] show that, even if a straight path tend to branch depending on the rupture velocity and the prestress orientation, the stress fields never allow obtuse branching angles as would be inferred from our field case. The mechanism illustrated in Figure 3(a) is not expected to occur in nature.

The detailed mapping of slip in the vicinity of the Kickapoo to Homestead Valley

Figure 3: Mechanisms possible to explain the "backwards propagation":(a)turn of rupture path through obtuse angle while continuing on main fault,(b) and (c)jumping mechanism;nucleation of bilateral rupture on neighboring fault

transition, Figure 2 ([25]) shows that there is no connection of slip on the two faults at the surface. Further, Li et al in [16] used studies of fault zone trapped waves to show that there was transmission in a channel along the Homestead Valley fault, and in another channel along the Southern Johnson Valley and Kickapoo faults, but no communication between those channels. Besides, Felzer and Beroza in [7] show through aftershocks relocations that the fault surface geometry (showed in the Figure 2) continues to the seismogenic depth. Those results suggest that the large-angle branching very likely occurred by a jumping mechanism illustrated in Figure 3(b) or (c). The mechanism (b) is unlikely: the stress concentration radiated from a propagating rupture tip is unlikely to cover a spatial extent required for nucleation along an adjoining fault. Indeed, similar to what we will notice in our dynamic study, the Harris and Day study in[10] elucidates this point. In the same way but using an energy analysis, Fossum and Freund in [8] show that the crack releases more energy per unti area (which is associated with stronger stress concentration) to the body when it slows abruptly.

The goal of this paper is then to study jumping following by a bilateral rupture propagation as a mechanism of backward branching. The earliest studies of fault steps are all 2-D quasi static analyses which do not include the time dependence of fault rupture. But a dynamic analysis is required to investigate the fault interactions and the rupture propagation through fault steps. Harris and Day in [10] describe dynamic rupture transfer between parallel offset fault segments. They use a two-dimensional finite difference computer program to study the effect of fault steps on dynamic ruptures. We want to widen their study to faults with more complex geometry: we begin simulating rupture propagation along parallel faults and then progress to curved faults. Such modelling, at least in 2D, is possible with the Boundary Integral Equation Method. With this, a new general mechanism of a jump followed by a bilateral propagation, leading to fault segments rupturing backwards by comparison with overall direction of developing rupture, can be studied numerically. As a first field example of such mechanism we study the rupture propagation from the Johnson Valley fault through the Kickapoo to Homestead Valley during the Landers 1992 earthquake.

There are different types of faults which correspond to the different mode of rupture. Our study focuses on the mode II rupture for vertical strike slip faults. So we assume that the fault surfaces are in contact everywhere, that is, there is tangential slip along the fault only and no opening. The right lateral or left lateral character for these faults give the direction of the slip of each side of the fault (Figure 4). We study here right lateral vertical strike slip faults and it is easy to generalized to the left lateral ones.

Figure 4: 2-D representation of right lateral vertical strike slip faults loaded by a initial stress field. When rupture propagates from left to right such that two of the fault segments are slipping at the same time, a right step is a dilational step and a right step is a compressional step.

We study the rupture propagation along faults in an infinite homogeneous isotropic elastic 2-D medium. The elastic properties of the material are time invariant. This study deals with a simple case and therefore makes a certain number of assumptions. But this is the first time that a numerical study shows that jumping mechanism is a way to explain backward branching.

Part I

Preliminary study

Chapter 1

Elastostatic singular crack solutions

The goal of this chapter is to give a general idea of the stress distribution near a crack and to begin to determine conditions so that a rupture can jump from one fault to another, parallel or not. For simplicity, we start with the study of an elastostatic singular crack model of a mode II shear rupture.
We consider here Coulomb friction stresses, stresses which oppose the motion or the tendency of motion between surfaces in contact. These stresses are proportional to the normal stress applied on the surface considered. When friction opposes motion in progress, it is the dynamic friction, $-\mu_d \left|\sigma_{normal}\right|$ and when it prevents a motion from initiating, it is the static friction, $-\mu_s \left|\sigma_{normal}\right|$.

We suppose that the two sides of the fault have finished their motion but all along the crack, there is sustained a uniform residual shear stress, $\mu_d \left|\sigma_{normal}\right|$ (as represented on Figure 1.1.1. This static study can be understood as a study after the motion. Different cases dealt by Harris and Day in [10] are first studied and finally we apply this analysis to the faults involved in the 1992 Landers earthquake.

1.1 Modelling and theoretical results

1.1.1 Stress distribution

We work here with a single straight crack extending from -X to 0 on the x-axis of the infinite x-y plane, in a mode II configuration and we study the stress distribution near the crack tip x=0.
The pre-stress has the form:

$$\sigma_{ij}^0 = \begin{pmatrix} \sigma_{xx}^0 & \sigma_{xy}^0 \\ \sigma_{xy}^0 & \sigma_{yy}^0 \end{pmatrix}. \tag{1.1}$$

As explained in Appendix A, the final stress σ_{ij} is the sum of the initial stress σ_{ij}^0

Figure 1.1: Singular elastic crack model of a mode II shear static rupture. Stress state shown (left) far behind the tip and (right) far ahead.

and stress change $\Delta\sigma_{ij}$ and is given by:

$$\sigma_{ij} = \frac{K_{II}}{\sqrt{2\pi r}}\Sigma_{ij}(\theta) + \begin{pmatrix} \sigma^0_{xx} & \tau_r \\ \tau_r & \sigma^0_{yy} \end{pmatrix} + O(\sqrt{r}) \tag{1.2}$$

where (r, θ) are the polar coordinates (the origin is the crack tip. In the present case:

$$K_{II} = (\sigma^0_{xy} - \tau_r)\sqrt{\frac{\pi X}{2}}.$$

1.1.2 Conditions for the rupture

Basically, in the Coulomb friction model, a rupture can nucleate at any point if the shear stress is higher that the static friction force. So, it is relevant to consider the normal and tangential stresses $(\sigma_{22}, \sigma_{21})$ at a point on a potential fault, whose polar coordinates are (r, θ). Different orientations given to the second fault are analyzed, Figure 1.2.

Figure 1.2: Study of the stress field for each point (r, θ) on a potential second fault which makes an angle ω with the initial fault.

Different situations of nucleation may arise:

1. If $\sigma_{21} > 0$ (right lateral slip), the rupture nucleates where $\sigma_{21} > \mu_s(-\sigma_{22})$ and when the normal stress is compressive ($\sigma_{22} < 0$). The area where this condition is verified is represented in red when the normal stress is compressive ($\sigma_{22} < 0$).

2. If $\sigma_{21} < 0$ (left lateral slip), it nucleates where $\sigma_{21} < -\mu_s(-\sigma_{22})$ and when the normal stress is compressive ($\sigma_{22} < 0$). The area where the condition is verified is represented in green. Here, it is interesting to know if there are areas where the left lateral slip is encouraged and what conditions allow such situation.

3. Finally, when the normal stress is extensional, ($\sigma_{22} > 0$). The concerned area is represented in blue. Generally, compressional stress fields are only studied so that the faults remained closed but it would be interesting to test if there are areas where the normal stress is extensional.

With these different representations, we are going to analyze where a nucleation can occur because of the presence of the crack. It is interesting without considering the dynamic process of the fracture to introduce the influence of different parameters: characteristics of the step (width and overlap), orientation of the faults, pre-stress, stress drop $\sigma^0_{yx} - \tau_r$, ratio $S = (\tau_p - \sigma^0_{yx})/(\sigma^0_{yx} - \tau_r)$ where $\tau_p = \mu_s \left|\sigma^0_{yy}\right|$ (peak strength)and $\tau_r = \mu_d \left|\sigma^0_{yy}\right|$ (residual stress),...

1.2 Influence of the different parameters: Harris and Day's cases

To study the influence of physical parameters, Harris and Day in [10] analyze cases with different values of $S = (\tau_p - \sigma^0_{yx})/(\sigma^0_{yx} - \tau_r)$, μ_s and μ_d defined in the table 1.1, for a fault of 28 km. σ^0_{xx} is not specified in 10 because only faults parallel to the

	Case A	Case B	Case C	Case D
$-\sigma^0_{yy}$	333 bars			
σ^0_{xx}	not precised			
σ^0_{yx}	200 bars			
μ_s	0.75	0.645	1.10	0.75
μ_d	0.3	0.51	0.3	0.51
S	0.49	0.49	1.65	1.65

Table 1.1: Different cases simulated

x-axis are studied, so it has no influence on the shear stress and the normal stress for parallel faults.

1.2.1 Study of parallel faults for each case

Figure 1.3: Representation of the points near a crack tip where a nucleation of a rupture is possible(shear stress above the peak strength) on a parallel fault for different conditions. The right lateral slip with a compressionnal normal stress is represented in red, the left lateral slip with a compressionnal normal stress in green and the extensional normal stress in blue.

From these stress distributions, we see that a simple static analysis is consistent with some conclusions of Harris and Day's paper.
First, is the difference between the compressional and the dilational sides. Indeed, the maximal "jumpable" distance is higher on the dilational side than on the compressional one. For example, for the case A, it is about 2km for the first side and 1.5 km for the other one. Besides, the critically stressed overlap distance can be only positive for the compressional side but positive and negative for the dilational one. Moreover, there is no symmetry, that is to say that the areas of possible nucleation

and the maximal "jumpable" distance are very different according to the overlap. Comparing the case A and B (in the same way C and D) for which the S ratio is the same but the stress drop ($\sigma^o_{xy} - \tau_r$ is different(100 bars for the first one and 40 bars for the second one), the stress drop seems to have no great influence in this static study (the stress distributions are not greatly different). Even if only for high stress drop (case A and C) there are points near the crack tip on the dilational side where the normal stress is positive which means a possible opening of the secondary fault, strong conclusions cannot be drawn because this is a particularity near the crack tip. Because of the singularity of the stress near the crack tip which has no physical sense, this simple model cannot be use to analyze features near the tip.
On the contrary, the S ratio is a more obvious influence on the stress distribution and the possibility of the jump of the rupture. We will see later in the dynamic study the importance of this ratio on the rupture velocity. For the static representation, an approximation of the expression of the stress neglecting the $O(\sqrt{r})$ in the equation 1.2 leads to:

$$\frac{H_{max}}{X} \propto \left(\frac{1}{1+S}\right)^2 \text{ where } H_{max} \text{ is the maximal jumpable distance.}$$

where H_{max} is the maximum jumpable distance and the coefficient of proportionality depends on μ_s. Indeed, comparing the cases A and C with the same stress drop but different S, it is obvious that for a high S, the dimensions of the shape are smaller than for a lower S. These dimensions decrease when S increases.
Considering only the points where the normal stress is compressive and the slip right lateral, it is interesting to represent where the difference $\sigma_{12} - \mu_s\,|\sigma_{22}|$ is the highest for a given width (y in this model). We call this difference Coulomb shear stress and note it σ_{coul}. Where the nucleation has more chance to occur on one given parallel fault might be pointed out. The Figure 1.4 corresponds to the case A. The location

Figure 1.4: Representation of the maximum Coulomb shear stress, $\sigma_{coul} = \sigma_{12} - \mu_s\,|\sigma_{22}|$, in the area where the nucleation in a secondary parallel fault is possible. The parameters are those of the case A

of the maximum Coulomb shear stress is linear with x, the overlap. In fact, the stress can be represented by the (x, y) variables instead of the (r, θ):

$$\sigma_{coul} = \frac{\mathrm{Const}_1}{\sqrt{y}}.F\left(\frac{x}{y}\right) + \mathrm{Const}_2$$

For a given y, the maximums of $\sigma_{12} - \mu_s\,|\sigma_{22}|$ are calculated which correspond to the zeros of F'. Thus, the function F' must have only one zero, that is to say a unique value of x/y.

1.2.2 Study of different orientations given to the secondary fault

In Harris and Day's article [10], only parallel secondary faults are studied. However, the natural examples are not so simple. In a 2D-analysis, the faults are rarely parallel and instead show slight or even large curvature. To determine the influence of the fault orientation on the shear and normal stresses and finally on the possibility of a rupture nucleation on this secondary fault is the goal of this section. Here, several orientations of faults are studied for just one set of conditions: the case A.
For parallel faults, it does not matter what value one assigns to σ^0_{xx}. For more complex configuration of faults, it is essential to give a relevant and realistic value to σ^0_{xx}. Some conditions about the initial stress has to be respected for this choice, as represented in the Figure 1.5. To properly represent the in-plane pre-stress field(three

Figure 1.5: Mohr circle of the pre-stress: conditions required.

components), and if we normalize with $-\sigma^0_{yy}$,where for all the subsequent discussion we assume $\sigma^0_{yy} < 0$, we have to specify only two quantities. We choose here $\sigma^0_{yx}/(-\sigma^0_{yy})$ and $\sigma^0_{xx}/(-\sigma^0_{yy})$.
Large regions of the Earth simply cannot sustain tensile stresses, so the normal stress for every orientation of faults cannot be positive. To avoid tension in the pre-stress state, the condition is:

$$(\frac{\sigma^0_{yx}}{-\sigma^0_{yy}})^2 \leqslant \frac{\sigma^0_{xx}}{\sigma^0_{yy}}.$$

To make sure that the pre stress does not violate the failure conditions for any orientation ω: $\left|\sigma^0_{12}\right| < -\mu_s\sigma^0_{22}$, $\sigma^0_{xx}/\sigma^0_{yy}$ has to respect the limits:

$$(\frac{\sigma^0_{xx}}{\sigma^0_{yy}})_{min,max} = \left[1 + \sin^2(\phi_s) \pm 2\sin(\phi_s)\sqrt{1 - \frac{1}{\mu_s^2}(\frac{\sigma^0_{yx}}{\sigma^0_{yy}})^2}\right]\frac{1}{\cos^2(\phi_s)}$$

In the case A ($\mu_s = 0.75$ and $\phi_s = 36,7^o$) the condition is:

$$1.002 < \frac{\sigma^0_{xx}}{\sigma^0_{yy}} < 3.248.$$

For the simulations, the value$(\sigma^0_{xx})/(\sigma^0_{yy}) = 1.5$ is chosen (Figure 1.5). As the angle

Figure 1.6: Representation of the points of an oblique secondary fault where the nucleation is possible.

increases from zero, as the maximal "jumpable" distance reduces. The shear stress, σ^0_{21} decreases with the angle ω in comparison with the normal stress, σ^0_{22}. For

$\omega = 30^o$, a left lateral slip is possible on the dilational side and for greater angles, other simulations show that a nucleation of rupture can occur only in the dilational side with a negative shear stress(left lateral slip). For negative angles, the maximal distance is comparable to the one for the parallel fault. But for larger negative angles, the zone where the stress is tensile and the one where the slip is left lateral increase. In the both sides of the fault, the nucleation is possible but the dilational side seems to give more favorable conditions to the rupture.
Thus, this study shows that if a second fault is straight and makes a positive angle with the first (like in the backward branching), the rupture is unlikely to jump from the first branch to the second one. But if some parts of the second fault are parallel or make a negative angle, the rupture is possible according to their distance from the tip of the crack and can perhaps propagate along the second fault. Thus, the complexity of the geometry of a second fault (curved part, irregularities,...) may favor a nucleation along it.

1.3 Study of the faults involved in the 1992 Landers earthquake

1.3.1 Modelling of the faults and choice of the parameters

According to[25], the nucleation of this earthquake occurred on the Johnson Valley Fault. The rupture propagated first along that fault, then it branched to the dilational side onto the Kickapoo fault, with the angle $\omega = 30^o$. The rupture also continued a few kilometers on the main (Johnson Valley) fault. In the present cases, this type of branching is not studied (Kame and al in [14] have already focused on such branched faults, including that case Johnson Valley fault to Kickapoo fault).Thus, we will neglect the continuation along the main fault and consider Johnson Valley fault and Kickapoo fault as one, and only one, main fault, whose length is 15 km (of course it is longer but we do not want to allow crack lengths in a 2-D model which are much greater than the seismogenic thickness of the crust). The rupture from the Kickapoo segment progressed not just forward onto the northern stretch of the Homestead Valley fault, but also backwards, i.e.,SSE along the Homestead Valley fault([25], [26], [29]). To determine the stress distribution due to the crack for the singular static model, Johnson Valley fault and Kickapoo fault are represented just here by a straight fault of 15 km. The Figure 1.7 gives the simple modelling of the faults. Actually, the smallest distance between Kickapoo fault and Homestead Valley fault is few hundred meters (between 200 and 300m), [25], and the orientation of the closest part of the last fault is between 0^o and 10^o. Besides, here, $tan(\Phi_s) = \mu_s$

Figure 1.7: Simple modelling of the faults involved in the 1992 Landers earthquake.

is always set to 0.6. It is less clear what to take for $\tan(\Phi_d) = \mu_d$, or how reason-

able it is to regard it as actually constant at large earthquake slip, especially when the possibility of thermal weakening and fluidization can be considered. Values of $\mu_d/\mu_s = 0.8$ and 0.2 have been tested and the results do not show significant differences. Only the results for $\mu_d/\mu_s = 0.2$ will be shown because this is the value chosen for the dynamic study.
As it was signaled in the last section, to properly determine the in plane pre stress field around the faults and if all the stresses are normalized by $-\sigma^0_{yy}$, two quantities have to be specified. First, on the basis of inference of principal stress directions from microseismicity by Hardebeck and Hauksson [9], the principal stress direction around the faults is approximately 30^o east of north. On the other angle, the tangent direction to the Kickapoo fault is about north. Thus there is an approximately $\Psi = 30^o$ angle between the most compressive stress and the main fault,(Figure 1.7). The choice of the S ratio would give another quantity of stress: $(\sigma^0_{yx})/(-\sigma^0_{yy})$. Its influence in this study was explained in the last section: the larger it is, the smaller the maximal jumpable distance is. It has a great influence on the rupture propagation, [2] which will be explained in the later sections. $S = 1.3$ is set here. It leads to $\sigma^0_{yx}/(-\sigma^0_{yy}) = 0.3287$. With this two parameters, the condition to avoid static friction shear failure for all possible fault surface orientations, for a given Ψ, is:

$$\frac{\sigma^0_{yx}}{-\sigma^0_{yy}} < \frac{\sin(\Phi_s)\sin(2\Psi)}{1-\sin(\Phi_s)\cos(2\Psi)} \tag{1.3}$$

In this case, the condition is $\sigma^0_{yx}/(-\sigma^0_{yy}) < 0.60$.
Given these two parameters, we can calculate, the ratio $\sigma^0_{xx}/\sigma^0_{yy}$, using the Mohr circle, Figure 1.3.1, for example:

$$\frac{\sigma^0_{xx}}{\sigma^0_{yy}} = 1 + 2\frac{\sigma^0_{yx}/(-\sigma^0_{yy})}{tan(2\Psi})) \tag{1.4}$$

Here, $\sigma^0_{xx}/\sigma^0_{yy} = 1.3796$.

Figure 1.8: Relations between the different parameters defining the in-plane stress field.

1.3.2 Stress distribution for some surface orientations

Figure 1.9: Representation of the areas where a nucleation of a rupture is possible according to the orientations of the second fault. These angles are chosen in relation with the geometry of Homestead Valley fault.

Comparing the stress distribution calculations for several orientations which represent where the nucleation of a rupture is possible and the position of the Homestead Valley fault and its orientations, we can conclude that the rupture can jump from Kickapoo fault and can nucleate in several positions, Figure 1.9 (although this analysis cannot tell us which will nucleate first).

1.3.3 Allowance of long-range dynamic rupture propagation for at least some orientations

The pre stress field plays a role in the continuation of the rupture along a fault. If the rupture nucleated along a suitable direction, will the pre stress be consistent with an arbitrary amount of propagation along that direction?
This condition will be met for at least some orientations if some part of the Mohr Circle lies outside the cone of angle $2\Phi_d$, as represented in Figure 1.10. Typically,

Figure 1.10: Condition for the pre stress to favor the propagation of the rupture for some orientations.

$\sigma^0_{yx}/(-\sigma^0_{yy}) > \mu_d$ makes long range dynamic rupture possible along the part of Homestead Valley fault parallel to the x-axis. Besides, the condition to make it possible along the other part of the fault, with an orientation of, $\omega = 30^o$ is : $\sigma^0_{12} > -\mu_r\sigma^0_{22}$. Using the Mohr circle, in this case, it means that $\sigma^0_{yx}/(-\sigma^0_{yy}) > 0.122$. Thus the pre-stress field allows a long range dynamic rupture possible along the fault.

Part II

Elastodynamic study

Chapter 2

Model and Equations

How can we even apply two-dimensional models to 'real' faults. The 2-D model is not ideal but this problem had been studied before, by Harris and Day [10] and for parallel faults. This model can predict how some geometrical parameters such as step-over width, geometry of the faults (curvature, orientations,...), some mechanical parameters(initial stress field, friction coefficients,...) affect the jumping mechanism.

2.1 Representation of elastic fields generated by faulting

This theory is from [1] and [21].

The Navier equations of motion for a homogeneous and isotropic linear elastic solid are:

$$\mathrm{div}\sigma + \mathbf{f} = \rho \frac{\partial^2 \mathbf{u}}{\partial t^2} \tag{2.1}$$

The stress-strain relation for an isotropic material is:

$$\sigma_{ij} = \lambda \delta_{ij} \varepsilon_{kk} + 2\mu \varepsilon_{ij} \tag{2.2}$$

where λ and μ are the Lame constants. In terms of displacement field, this equation becomes:

$$(\lambda + \mu)\nabla(\nabla \cdot \mathbf{u}) + \mu \nabla^2 \mathbf{u} + f = \rho \frac{\partial^2 \mathbf{u}}{\partial t^2}, \tag{2.3}$$

or,

$$i\epsilon\{1,2,3\} \quad (\lambda + \mu)\frac{\partial}{\partial x_i}\frac{\partial u_j}{\partial x_j} + \mu \frac{\partial^2 u_i}{\partial x_j^2} + f_i = \rho \frac{\partial^2 u_i}{\partial t^2} \tag{2.4}$$

where $\mathbf{u}$ is the displacement field.

2.1.1 Green's function

$G_{ij}(\mathbf{x}, t)$ is the response to a concentrated impulsive force. That is, $G_{ij}(\mathbf{x}, t)$ is the solution for $u_i(\mathbf{x}, t)$ when the body force density is $f_k = \delta_{kj}\delta_{Dirac}(t)\delta_{Dirac}(\mathbf{x})$.

Operating on the equation (2.4) with the full space-time Fourier transform, we can find that the solution is the sum of two terms:

$$G_{ij} = G^p_{ij} + G^s_{ij}$$

where

$$\begin{cases} G^p_{ij} &= -\frac{\partial^2}{\partial x_i \partial x_j} h(r,t;c_p), \\ G^s_{ij} &= -(\nabla^2 \delta_{ij} - \frac{\partial^2}{\partial x_i \partial x_j}) h(r,t;c_s), \end{cases}$$

and,

$$c_p = \sqrt{\frac{\lambda + 2\mu}{\rho}}, \quad c_s = \sqrt{\frac{\mu}{\rho}}$$

define the dilational (P-wave) and shear (S wave) wave speeds.
h is given by:

$$h(r,t;c) = -\frac{1}{4\pi\rho r}(t - \frac{r}{c})U(t - \frac{r}{c}), \tag{2.5}$$

where, U is the unit step function, and this has the desirable property of making both contributions to G_{ij} vanish separately when $r > c_p t$. In fact, G_{ij} also vanishes for $r < c_s t$, has delta function singularities at $r = c_s t$ and $r = c_p t$ and has a variable structure with r between these outgoing fronts. For a complete expression of the Green functions see Appendix B.

2.1.2 Response to given distribution of displacement discontinuity

Moment density tensor

Typically, rupture is modelled as the development of a displacement discontinuity $\Delta\mathbf{u}(\mathbf{x},t)$ on a surface or collection of surfaces S. Rapid processes of fault slippage generate waves. This source of elastic displacement fields can generally be represented, kinematically, by distributions of *transformation strain* $\varepsilon^T_{ij}(\mathbf{x},t)$. The transformation strain describes an alteration of the *stress free* configuration of a solid. The usual relation between stress and strain for an isotropic material is altered to:

$$\sigma_{ij} = \lambda\delta_{ij}(\varepsilon_{kk} - \varepsilon^T_{kk}) + 2\mu(\varepsilon_{ij} - \varepsilon^T_{ij}) \tag{2.6}$$

where

$$\varepsilon_{ij} = \frac{1}{2}(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}) \tag{2.7}$$

This assumes no alteration of the elastic moduli due to the transformation strain. We introduce the notation:

$$m_{ij} = \lambda\delta_{ij}\varepsilon^T_{kk} + 2\mu\varepsilon^T_{ij} \tag{2.8}$$

where m_{ij} is called the moment density tensor.
Let now the sides of S be noted + and -, so that $\Delta\mathbf{u} = u^+ - u^-$, and let $\mathbf{n}$ be the

local unit normal to S, directed from $-$ to $+$. We can deal with this case as the zero-thickness limit of transformation strain in a thin layer coinciding with S; ε must be of order $\Delta\mathbf{u}$ divided by the layer thickness, and hence become Dirac singular in the limit. In particular, if δV is some small volume element intersected dy δS of the surface,

$$\int_{\delta v} \varepsilon_{ij}^T d^3\mathbf{x} = \frac{1}{2}(n_i \Delta u_j + n_j \Delta u_i)\delta S \tag{2.9}$$

Hence,

$$\varepsilon_{ij}^T = \frac{1}{2}(n_i \Delta u_j + n_j \Delta u_i)\delta_{Dirac}(S) \tag{2.10}$$

Double couple response

We search first the solution for a distribution of body force $f_k(\mathbf{x}, t)$ in the material that meets homogeneous boundary conditions (i.e. zero tractions on the Earth's surface). Because of the linearity of the equation 2.4, the Green's function $G_{ij}(\mathbf{x}, t)$ may then be defined as the weighting coeficient in such a linear response, so that we can write the solution to the equation of motion as:

$$u_i(\mathbf{x}, t) = \int_{-\infty}^{t} \int_V G_{ij}(\mathbf{x} - \mathbf{x}', t - t') f_j(\mathbf{x}', t) d^3\mathbf{x}' dt' \tag{2.11}$$

Then, we want to find the body force equivalent of a displacement discontinuity on S. When the stress-strain relation 2.6 is substituted into the equations of motion 2.1, with $f = 0$, there results equation for $\mathbf{u}$ in the same form as 2.4 but with $\mathbf{f}$ replaced by an "effective" body force

$$f_j^{eff} = -\frac{\partial m_{ij}}{\partial x_i} \tag{2.12}$$

We assume that the medium is at rest with no slip for time $t \leq 0$ and also that the traction is continuous accross the crack. Substitution of f_j^{eff} of 2.12 into 2.11 shows, after an integration by parts (in which we assume that $\varepsilon^{\mathbf{T}}$ vanishes on the surface of the body) and after considering the symmetry of $\varepsilon^{\mathbf{T}}$ and so of $\mathbf{m}$, that the displacement field generated by the displacement discontinuity is:

$$u_i(\mathbf{x}, t) = \frac{1}{2}\int_{-\infty}^{t} \int_V H_{ijk}(\mathbf{x} - \mathbf{x}', t - t') m_{jk}(\mathbf{x}', t') d^3\mathbf{x}' dt' \tag{2.13}$$

where

$$H_{ijk}(\mathbf{x} - \mathbf{x}', t - t') = \frac{\partial G_{ik}(\mathbf{x} - \mathbf{x}', t - t')}{\partial x_j'} + \frac{\partial G_{ij}(\mathbf{x} - \mathbf{x}', t - t')}{\partial x_k'} \tag{2.14}$$

is called the *double-couple response* when j and k differ and the *linear vector dipole response* when they agree. $H_{ijk}(\mathbf{x} - \mathbf{x}', t - t')$ can be interpreted as the displacement

$u_i(\mathbf{x}, t)$ at $\mathbf{x}$ in response to the application at $\mathbf{x}'$ of a pair of *impulsive force dipoles with zero net moment*. One such dipole is arrayed along the k-direction with its impulses in the j-direction, the other arrayed along the j-direction with its impulses in the k-direction. We may write H_{ijk} in terms of "p" and "s" contributions. Thus,

Figure 2.1: Double couples without moment: a)$u_i = H_{i12} = H_{i21}$, b) $u_i = H_{i11}$, c) and d) relation to release of constraint stresses.

using the representation for $u_i(\mathbf{x}, t)$ of 2.13, the expressions for H_{ijk} in terms of "d" and "s" contributions and the symmetry of the seismic moment density tensor, we now obtain the following results for the displacement field:

$$u_i(\mathbf{x}, t) = u_i^p(\mathbf{x}, t) + u_i^s(\mathbf{x}, t) \tag{2.15}$$

where,

$$\begin{cases} u_i^p = -\dfrac{\partial^3}{\partial x_i \partial x_j \partial x_k} \displaystyle\int_{-\infty}^{t} \int_V h(r, t-t'; c_p) m_{jk}(\mathbf{x}', t') d^3\mathbf{x}' dt', \\ u_i^s = -\dfrac{\partial}{\partial x_k} (\nabla^2 \delta_{ij} - \dfrac{\partial^2}{\partial x_i \partial x_j}) \displaystyle\int_{-\infty}^{t} \int_V h(r, t-t'; c_s) m_{jk}(\mathbf{x}', t') d^3\mathbf{x}' dt', \end{cases} \tag{2.16}$$

where, $r = |\mathbf{x} - \mathbf{x}'|$ and h given by 2.5.
If we take the limit of these expressions for r large compared to the wavelengths associated to c_p and c_s and to the dimensions of the source (the fault), we could observe that u_p is in the direction of $\mathbf{x}$, where as u_s is perpendicular to $\mathbf{x}$ and that there is time delays due to "p" and "s" wave propagation between motions in the source region and the reception of their effects at the receiver point.

Seismic displacement in terms of a causative displacement discontinuity across a fault in a 2-D medium

With the expression 2.13, we can apply the dynamic representation theorem to a crack in an infinite homogeneous isotropic elastic 2-D medium and express the elastic displacement field:

$$u_i(\mathbf{x}, t) = \frac{1}{2} \int_{-\infty}^{t} \int_{\Gamma} H_{ijk}(\mathbf{x} - \mathbf{x}', t - t') m_{jk}(\mathbf{x}', t') d^3\mathbf{x}' dt' \tag{2.17}$$

where all the notations are explained in Figure 2.2. Here, the kernel function H_{ijk}

Figure 2.2: Nomenclature used in the crack analysis.

contain hypersingularities and we have to take a finite part of the divergent integral; the details will be given in Appendix C. The displacement discontinuity on the crack has only a tangential component (purely mode II) Δu_t. We obtain the relations:

$$\begin{cases} \Delta u_1(\mathbf{x}', t) = n_2(x')\Delta u_t(\mathbf{x}', t), \\ \Delta u_2(\mathbf{x}', t) = -n_1(x')\Delta u_t(\mathbf{x}', t) \end{cases} \tag{2.18}$$

Using the above notations we have:

$$\begin{aligned} u_1(\mathbf{x}, t) = &-\text{p.f.} \int_\Gamma d\mathbf{x}' \int_0^t dt' \left\{ [\Delta u_1(\mathbf{x}', t')(\lambda + 2\mu)n_1(\mathbf{x}') + \Delta u_2(\mathbf{x}', t')\lambda n_2(\mathbf{x}')] \frac{\partial G_{11}}{\partial x_1} \right. \\ &+ [\Delta u_1(\mathbf{x}', t')\lambda n_1(\mathbf{x}') + \Delta u_2(\mathbf{x}', t')(\lambda + 2\mu)n_2(\mathbf{x}')] \frac{\partial G_{12}}{\partial x_2} \\ &\left. + [\Delta u_1(\mathbf{x}', t')\mu n_2(\mathbf{x}') + \Delta u_2(\mathbf{x}', t')\mu n_1(\mathbf{x}')] \left(\frac{\partial G_{11}}{\partial x_2} + \frac{\partial G_{12}}{\partial x_1} \right) \right\} \\ = &-\text{p.f.} \int_\Gamma d\mathbf{x}' \int_0^t dt' \Delta u_t(\mathbf{x}', t')\mu \\ &\times \left\{ 2n_1(\mathbf{x}')n_2(\mathbf{x}') \left(\frac{\partial G_{11}}{\partial x_1} - \frac{\partial G_{12}}{\partial x_2} \right) + [n_2^2(\mathbf{x}') - n_1^2(\mathbf{x}')] \left(\frac{\partial G_{11}}{\partial x_2} + \frac{\partial G_{12}}{\partial x_1} \right) \right\} \\ = &-\text{p.f.} \int_\Gamma d\mathbf{x}' \int_0^t dt' \Delta \dot{u}_t(\mathbf{x}', t')\mu \\ &\times \left\{ 2n_1(\mathbf{x}')n_2(\mathbf{x}') \left(\frac{\partial R_{11}}{\partial x_1} - \frac{\partial R_{12}}{\partial x_2} \right) + [n_2^2(\mathbf{x}') - n_1^2(\mathbf{x}')] \left(\frac{\partial R_{11}}{\partial x_2} + \frac{\partial R_{12}}{\partial x_1} \right) \right\}, \end{aligned} \tag{2.19}$$

$$\begin{aligned} u_2(\mathbf{x}, t) = &-\text{p.f.} \int_\Gamma d\mathbf{x}' \int_0^t dt' \Delta \dot{u}_t(\mathbf{x}', t')\mu \\ &\times \left\{ 2n_1(\mathbf{x}')n_2(\mathbf{x}') \left(\frac{\partial R_{12}}{\partial x_1} - \frac{\partial R_{22}}{\partial x_2} \right) + [n_2^2(\mathbf{x}') - n_1^2(\mathbf{x}')] \left(\frac{\partial R_{12}}{\partial x_2} + \frac{\partial R_{22}}{\partial x_1} \right) \right\}, \end{aligned} \tag{2.20}$$

where the displacement field is represented in terms of the slip velocity $\dot{u}_t(\mathbf{x}', t')$ by way of integration by parts in terms of t'. The Green's function is then integrated to give $R_{ij}(\mathbf{x}', t') = \int_0^{t'} G_{ij}(\mathbf{x}', s)ds$ which is the response to a Heaviside step force applied at $\mathbf{x}'$ at time t'. Hypersingularities disappear in the equations 2.19 and 2.20 as a result of the integration.

Actually, using the characteristics of the Green's functions pointed out in the beginning of this section, a point cannot receive the information of slip of every point along the fault. We have to consider the proprieties of the wave propagation cone (defined by the P-wave velocity) to reduce the intervals of integrations (Figure 2.3. Hence for all the integrals below, we have just to replace the actual time t by the time $\tau_m = max(0, t - \|\mathbf{x} - \mathbf{x}'\|/c_p)$

Figure 2.3: Domain of integration of the integral equation: a point can receive information of all the other points that lie inside the causality cone defined by the P-wave velocity c_p.

Stress field in terms of a causative displacement discontinuity across a fault in a 2-D medium

The application of the stress-strain relation 2.2 leads to the following representation for stresses:

$$\frac{1}{2}[\sigma_{11}(\mathbf{x},t)-\sigma_{22}(\mathbf{x},t)] = -\text{p.f.}\int_\Gamma d\mathbf{x}'\int_0^{\tau_m} dt'\Delta\dot{u}_t(\mathbf{x}',t')\mu^2 \times\Big\{[n_2^2(\mathbf{x}')-n_1^2(\mathbf{x}')]\left[\frac{\partial^2}{\partial x_1\partial x_2}(R_{11}-R_{22})+(\frac{\partial^2}{\partial x_1^2}-\frac{\partial^2}{\partial x_2^2})R_{12}\right] +2n_1(\mathbf{x}')n_2(\mathbf{x}')\left(\frac{\partial^2}{\partial x_1^2}R_{11}+\frac{\partial^2}{\partial x_2^2}R_{22}-2\frac{\partial^2}{\partial x_1\partial x_2}R_{12}\right)\Big\}, \tag{2.21}$$

$$\frac{1}{2}[\sigma_{11}(\mathbf{x},t)+\sigma_{22}(\mathbf{x},t)] = -\text{p.f.}\int_\Gamma d\mathbf{x}'\int_0^{\tau_m} dt'\Delta\dot{u}_t(\mathbf{x}',t')\mu(\lambda+\mu) \times\Big\{[n_2^2(\mathbf{x}')-n_1^2(\mathbf{x}')]\left[\frac{\partial^2}{\partial x_1\partial x_2}(R_{11}+R_{22})+(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2})R_{12}\right] +2n_1(\mathbf{x}')n_2(\mathbf{x}')\left(\frac{\partial^2}{\partial x_1^2}R_{11}-\frac{\partial^2}{\partial x_2^2}R_{22}\right)\Big\}, \tag{2.22}$$

$$\sigma_{12}(\mathbf{x},t) = -\text{p.f.}\int_\Gamma d\mathbf{x}'\int_0^{\tau_m} dt'\Delta\dot{u}_t(\mathbf{x}',t')\mu^2 \times\Big\{[n_2^2(\mathbf{x}')-n_1^2(\mathbf{x}')]\left[\frac{\partial^2}{\partial x_2^2}R_{11}+\frac{\partial^2}{\partial x_1^2}R_{22}+2\frac{\partial^2}{\partial x_1\partial x_2}R_{12}\right] +2n_1(\mathbf{x}')n_2(\mathbf{x}')\left(\frac{\partial^2}{\partial x_1\partial x_2}(R_{11}-R_{22})+(\frac{\partial^2}{\partial x_1^2}-\frac{\partial^2}{\partial x_2^2})R_{12}\right)\Big\}, \tag{2.23}$$

The slip velocity is directly associated with a traction change on a crack, like in the study of Cochard and Madariaga for anti-plane shear crack, in [3]. Here, the integrals representations (equations 2.21, 2.22 and 2.23) contain hypersingularities that cannot be discretized directly (see Appendix C).
We obtain for the shear traction $T_t(\mathbf{x},t)$ on the crack:

$$T_t(\mathbf{x},t) = n_1(\mathbf{x})n_2(\mathbf{x})[\sigma_{11}(\mathbf{x},t)-\sigma_{22}(\mathbf{x},t)]+[n_2^2(\mathbf{x}')-n_1^2(\mathbf{x}')]\sigma_{12}(\mathbf{x},t) \tag{2.24}$$

and for the normal traction:

$$T_n(\mathbf{x},t) = [\sigma_{11}(\mathbf{x},t)+\sigma_{22}(\mathbf{x},t)]-[n_2^2(\mathbf{x}')-n_1^2(\mathbf{x}')][\sigma_{11}(\mathbf{x},t)-\sigma_{22}(\mathbf{x},t)] + 2n_1(\mathbf{x})n_2(\mathbf{x})\sigma_{12}(\mathbf{x},t) \tag{2.25}$$

The 2-D Green's functions are given in Appendix B.

2.1.3 Stress field representation for a static slip

In the static problem, the representation theorem for the displacement and stress fields does not include the time. By analogy to the last equations, we find:

$$\begin{aligned}\frac{1}{2}[\sigma_{11}(\mathbf{x})-\sigma_{22}(\mathbf{x})] = -\text{p.f.}\int_\Gamma & d\mathbf{x}'\Delta u_t(\mathbf{x}')\mu^2 \\ &\times\left\{[n_2^2(\mathbf{x}')-n_1^2(\mathbf{x}')]\left[\frac{\partial^2}{\partial x_1\partial x_2}(G_{11}-G_{22})+(\frac{\partial^2}{\partial x_1^2}-\frac{\partial^2}{\partial x_2^2})G_{12}\right]\right. \\ &\left.+2n_1(\mathbf{x}')n_2(\mathbf{x}')\left(\frac{\partial^2}{\partial x_1^2}G_{11}+\frac{\partial^2}{\partial x_2^2}G_{22}-2\frac{\partial^2}{\partial x_1\partial x_2}G_{12}\right)\right\},\end{aligned} \tag{2.26}$$

$$\begin{aligned}\frac{1}{2}[\sigma_{11}(\mathbf{x})+\sigma_{22}(\mathbf{x})] = -\text{p.f.}\int_\Gamma & d\mathbf{x}'\Delta u_t(\mathbf{x}')\mu(\lambda+\mu) \\ &\times\left\{[n_2^2(\mathbf{x}')-n_1^2(\mathbf{x}')]\left[\frac{\partial^2}{\partial x_1\partial x_2}(G_{11}+G_{22})+(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2})G_{12}\right]\right. \\ &\left.+2n_1(\mathbf{x}')n_2(\mathbf{x}')\left(\frac{\partial^2}{\partial x_1^2}G_{11}-\frac{\partial^2}{\partial x_2^2}G_{22}\right)\right\},\end{aligned} \tag{2.27}$$

$$\begin{aligned}\sigma_{12}(\mathbf{x}) = -\text{p.f.}\int_\Gamma & d\mathbf{x}'dt'\Delta u_t(\mathbf{x}')\mu^2 \\ &\times\left\{[n_2^2(\mathbf{x}')-n_1^2(\mathbf{x}')]\left[\frac{\partial^2}{\partial x_2^2}G_{11}+\frac{\partial^2}{\partial x_1^2}G_{22}+2\frac{\partial^2}{\partial x_1\partial x_2}G_{12}\right]\right. \\ &\left.+2n_1(\mathbf{x}')n_2(\mathbf{x}')\left(\frac{\partial^2}{\partial x_1\partial x_2}(G_{11}-G_{22})+(\frac{\partial^2}{\partial x_1^2}-\frac{\partial^2}{\partial x_2^2})G_{12}\right)\right\},\end{aligned} \tag{2.28}$$

where G_{ij} are the elasto-static Green's functions given in the Appendix B.

2.2 Elastic crack modelling: Slip-weakening Coulomb friction law

In models often presented in the literature, the rupture velocities are predetermined and fixed. In this modelling, the rupture was allowed to propagate spontaneously using a slip-weakening friction law originally proposed by Ida [11] and Palmer and Rice [18]. The fault strength τ, once reaching the peak strength τ_p, decreases linearly 9in the most commonly adopted variant of slip-weakening) with the slip to the residual strength τ_r and becomes constant when the slip Δu exceeds an amount D_c, the slip weakening critical distance. D_c is considered to be a parameter inherent in the rupture process. Moreover, the Coulomb friction law is added to the slip-weakening law which is widely used to study friction on faults. Then, τ is proportional to the normal stress $-\sigma_n$ at any particular amount of slip, as see Figure 2.4

$$\tau = \tau_r + (\tau_p - \tau_r)(1 - \frac{\Delta u}{D_c})H(1 - \frac{\Delta u}{D_c}) \tag{2.29}$$

where

$$\tau_p = \mu_s \times (-\sigma_n) \qquad \tau_r = \mu_d \times (-\sigma_n) \tag{2.30}$$

This criterion, contrary to the critical stress intensity factor criterion, does not

Figure 2.4: Slip-weakening friction Coulomb law. The peak and residual strength (τ_p,τ_r), and strength (τ) at any particular amount of slip (Δu), is proportional to normal stress ($-\sigma_n$).

suffer the unphysical infinite stresses at the edges: there is a continuous stress distribution at the crack tip. (see Figure 2.5). The notation R denotes the size of the slip-weakening zone, i.e. the zone in which $0 < \Delta u < D_c$ and $\sigma_{21} > \tau_r$.

Figure 2.5: Fault and distribution of shear stress σ_{21} and slip displacement Δu.

Rupture is allowed along the pre-existing faults where these frictional proprieties are assumed to be uniform over the whole fault system. But their length are fixed so in this model that corresponds to a very high static friction coefficient so that rupture could not break through into the surrounding medium.

2.2.1 Estimation of the parameters of the criterion

Energy balances

First, Palmer and Rice in [18] have determined the energy surplus available per unit length of advance of the crack. This surplus is the excess of the work input of the applied forces (here they are the forces equivalent to the displacement discontinuities) over the sum of the energy absorbed in deforming material outside the crack and the frictional dissipation against the residual part τ_r of the slip resistance within the crack.

$$\mathsf{J} = \int_\Gamma [\sigma(\Delta u) - \tau_r] d(\Delta u) \tag{2.31}$$

In the slip weakening law chosen for our model, it leads to :

$$\boxed{\mathsf{J} = \frac{1}{2}(\tau_p - \tau_r)D_c} \tag{2.32}$$

For propagation to occur, this energy surplus must balance the additional dissipation in the end region (slip weakening zone) against shear strengths in excess of the residual.
If the length of the end region is infinitely small, along the entire length of the crack the shear stress is equal to the residual strength and the stresses become infinite at the tip the cracks. This limit corresponds to the linear elastic crack criterion. In static conditions in this criterion, Rice in [21] determines the energy $I\delta l$ needed to the crack front to advance infinitesimally by δl and define the fracture energy I:

$$I = \frac{3}{8\mu}K^2 \tag{2.33}$$

where K is the stress intensity factor and with a Poisson ratio equal to 0.25.
Rice in [20] and [21] showed that if the length of the slip weakening zone, R_0, is small compared to all geometric dimensions of the model, such as overall crack size,the propagation criterion is:

$$\underbrace{I}_{\text{energy release rate due to the crack}} = \underbrace{\mathtt{J}}_{\text{energy needed for the propagation}} \tag{2.34}$$

that is

$$\frac{3}{8\mu}K^2 = \frac{1}{2}(\tau_p - \tau_r)D_c \tag{2.35}$$

where K is determined from an analysis which neglects the end zone.

Minimum nucleation size

Using the equation 2.35, we can estimate the minimum nucleation size of an initial crack so that with the slip weakening law the rupture can propagate:

$$\boxed{L_c = \frac{8}{3\pi}\frac{\mu(\tau_p - \tau_r)}{(\sigma^0_{xy} - \tau_r)^2}D_c} \tag{2.36}$$

So the initial crack has to be long enough to permit the rupture to propagate along the fault but has to be small compared to the fault length to not affect the dynamic results.

Estimate of the size of the slip weakening zone

Palmer and Rice in [18] and Rice in [21]use another slip weakening law in which σ varies linearly with x_1 within the end zone. If the end zone size is very small in comparison to the other lengths such as the crack length and the minimum nucleation size, they determine an exact expression for R_0. In our model, we cannot calculate an exact value of the static end zone size. The results of [18] and [21] is quite realistic and is often used as estimation of the static end zone size of our slip weakening law:

$$\boxed{R_0 = \frac{3\pi}{8} \frac{\mu}{\tau_p - \tau_r} D_c} \tag{2.37}$$

Rice in [21] pointed out that for the same slip weakening law, during the propagation, the dynamic end zone size R is a function of the rupture velocity and diminishes with the velocity in particular way. That is:

$$R = \frac{R_0}{f(v_r)} \tag{2.38}$$

where $f = 1$ when $v_r = 0^+$ and increase with v_r, without limit as $v_r \rightarrow c_R$, with in this model, $c_R = 0.9194c_s$ is the Raileigh wave speed.

2.2.2 Influence of S in the rupture velocity

In lots of models presented in the literature, the rupture velocities are predetermined. Here, the crack propagates spontaneously according to the slip weakening Coulomb fracture criterion: the rupture velocity is not fixed.
Andrews in[2] and Das and Aki in [4] the rupture velocity depends on the relationship between the the stress increase to initiate slip and the final stress drop if the crack length and the initial nucleation size is fixed. They define a dimensionless parameter S to quantify this ratio we used in the part I:

$$\boxed{S = \frac{\tau_p - \sigma_{xy}^0}{\sigma_{xy}^0 - \tau_r}} \tag{2.39}$$

. Using the Figure 9 of [2] and the Figure 15 of [4], according to the crack length and the initial nucleation size, the rupture velocity $v_r$can be lower (sub-shear) or higher (super-shear) than the shear wave velocity c_s.

Chapter 3

Numerical modelling of dynamic rupture: Boundary Integral Equation Method

Most of the numerical studies of dynamic crack propagation were restricted to a prescribed straight crack path. However real cracks present more complex geometry: bends and bifurcations, branches, multiple strands and stepovers... Elastodynamic analysis of cracks of non-planar geometry has been studied only recently. Three different numerical approaches have been used. One of them, the finite difference method (FDM) (used in [2], [10] and [12] for example) discretizes the equations of motion representing the model domain by a regularly spaced rectangular grid system. The problems of this method as discussed by Das and Kostrov in [5] the resolution of stress near the crack tip, the numerical dispersion and the necessity to solve the equations all over the model. A second, the finite element method discretized, too, the complete model domain but it is divided into a mesh of element of simple shape that need not to be regularly spaced. Though this method can be applied theoretically to non-planar cracks, remeshing procedures reveal significant difficulty. The last one is the Boundary integral equation method (BIEM) (4], [13], [14], [24] for example) which solves integral equations that relates the slip on the crack with the stress on the crack; the rest of the domain is not explicitly concerned in the formulation. The advantage of this method is a reduced dimension of the problem and an easy representation of arbitrarily curved cracks.
We want to discretize the equations 2.19, 2.20, 2.21, 2.22, 2.23 using the slip weakening Coulomb friction law 2.29 as boundary conditions.

3.1 Discretization of the equation

The equations are discretized using the technique developed by Koller and al. in [15] and Cochard and Madariaga in [3].

3.1.1 Implementation of the space time domain

To implement the space time domain of integration of the equations, the fault is approximated by a polygon consisting of m elements of constant length Δs : $(x'_1, x'_2, ..., x'_m)$ are the end points with curvilinear abscisse : $(s_1, s_2, ..., s_m)$. The time is also discretized by a set of equally spaced time steps with an interval of Δt. The slip velocity is approximated by a linear combination of a set of basis functions that are assumed to be constant over an element and discontinuous between elements, this is the piecewise constant interpolation ([3], [14], [13]):

$$\Delta \dot{u}(s,t) = \sum_i \sum_k V^{k,l} \phi^i(s) \theta^k(t) \tag{3.1}$$

with

$$\phi^i(s) = \begin{cases} 1 & \text{if } s^i < s < s^{i+1} \\ 0 & otherwise \end{cases} \tag{3.2}$$

$$\theta^k(t) = \begin{cases} 1 & \text{if } t^{k-1} < t < t^k \\ 0 & otherwise, \end{cases} \tag{3.3}$$

where ϕ^i and θ^k are the spatial and temporal interpolation functions $s^i = i\Delta s$ and $t^k = k\Delta t$, and $V^{i,k}$ is the discretized slip velocity at the kth time step on the ith element.

The nodal points or collocation points $(y_1, y_2, ..., y_m)$are used to evaluate the stress for the boundary condition and for the fracture criterion. Reasoning with the regularization procedure, utilized in the BIEM formulation and the symmetry of certain expressions, Koller and al. in [15] and Cochard and Madariaga in [3] showed that the nodal points in space had to be the middle of each cells: their curvilinear abscisse $(s^j)' = (s^j + s^{j+1})/2$. They found too that these calculations should be done at the time $t_n = n\Delta t$ to produce the best results.

In [3], [15], [13] among others, the ratio $c_p \Delta t / \Delta s$ has been chosen equal to 1/2. This value is smaller than $1/\sqrt{2}$, and therefore respects the stability conditions of corresponding two-dimensional finite difference methods, as explained in [15].

3.1.2 Discretization of the time-domain equations

Substituting the interpolated slip velocity distribution given in the equation 3.1, into the stress representation, eq. 2.21, 2.22, 2.23, the discrete form of the stress components is:

$$\sigma_{ij}(\mathbf{x}, t^n) = \sum_{k=1}^{n} \sum_i V^{i,k} I^{i,k-n}(\mathbf{x}) \tag{3.4}$$

$I^{i,k-n}(\mathbf{x})$ is the discretized stress operator, which represents the contribution of a unit slip velocity of the cell i at the time step n-k to the stress at the observation position $\mathbf{x}$ and time t^n. It is expressed thanks to the functions $\partial^2 R_{ij}/\partial x_p \partial x_q$ involved in the equations 2.21, 2.22, 2.23:

$$I^{i,k-n}(\mathbf{x}) = \text{p.f.} \int_{s^i}^{s^{i+1}} \int_{t^{k-1}}^{t^k} F(s,t')ds dt' \tag{3.5}$$

where F is the part of the integrand multiplying the slip rate in those expressions. We have mentioned in the section 2.1.2 that the convolutions have to be calculated inside the wave cones which involve hypersingularities at the wave fronts. However, the definition of the finite part introduced in [13] and explained in the Appendix C enables the evaluation of this hypersingular integral $I^{i,k-n}(\mathbf{x})$ analytically.
Using the expressions 2.24 and 2.25 of the tangential and normal tractions respectively, due to the distribution of displacement discontinuities along the fault (without considering the initial stress) from the stress components of the equation 3.4, the boundary integral equations are reduced to a set of linear algebraic equations:

$$T_t^{l,n} = K_t^{0,0} V^{l,n} + \sum_{k=1}^{n} \sum_i V^{i,k} K_t^{l-i,n-k}, \tag{3.6}$$

$$T_n^{l,n} = \sum_{k=1}^{n} \sum_i V^{i,k} K_n^{l-i,n-k}, \tag{3.7}$$

The traction is evaluated at time t^n and at the nodal point: $(s^l)' = (s^l + s^{l+1})/2$.

x

Figure 3.1: Schematic diagram of the discretized BIEM. The points represents the cells with non zero slip velocity

$K_t^{0,0} = -\mu/(2c_s)$ is the radiation damping term [3] that represents the instantaneous contribution of the current slip velocity to the shear stress at the same position. The second terms on the right hand side of the two expressions contains the contribution of the previous slip velocities. $K_n^{0,0}$ is zero because we consider no opening along the fault, so the current slip velocity does not contribute to the calculation of the normal stress. $K^{l-i,n-k}$ represents the stress at the center of cell l, at the end of time step n due to a unit slip velocity within cell i during time step k.
Actually, because of the proprieties of the wave propagation cone represented in the Figure 2.3, we can reduce the number of time steps and cells whose slip velocity contributes really to the calculations of stress, as explained in the Figure 3.1. The convolution has to be done only for those spatio-temporal elements that fall inside the wave cones.
To determine $T_t^{l,n}$ and $V_{l,n}$ at a current time step, we solve the equation 3.6 under an imposed boundary conditions

3.1.3 Discretization of the time independent equations

Following the same symbolic notations and using the equations 2.26, 2.27 and 2.28, applying a discretization where a constant slip D^i is assumed within each spatial cell, we can write the discrete form of the stress components and then of the tangential and normal stress on the cell l, as in the last section:

$$T_t^l = \sum_i D^i K_{t,static}^{l-i}, \tag{3.8}$$

$$T_n^l = \sum_i D^i K_{n,static}^{l-i}, \tag{3.9}$$

where l and i represent the discretized position on the fault. The term $K_{t,static}^{l-1}$ (or $K_{n,static}^{l-1}$) is the static stress kernel that indicates the shear (or the normal) stress at element l due to unit slip at element i.

3.2 The Program

Our numerical computations are made using the following non-dimensional quantities:

1. the time is normalized by $\Delta s/2c_p$
2. all the lengths, except the slip, are normalized by Δs
3. slips are normalized by $\Delta s(-\sigma_{yy}^0)/\mu$
4. velocities are normalized by $c_p(-\sigma_{yy}^0)/\mu$
5. stresses are normalized by $(-\sigma_{yy}^0)$

The difficult step of the implementation is how to choose Δs according to our model. We have chosen a slip weakening Coulomb friction law. The size of the zone where these conditions are influent decreases from a static value R_0, defined in the last chapter to 0 as the rupture velocity increases. The smaller Δs is, the longer is the time when the law is properly represented. Once the size of the end zone R is too small, the model is reduced to a singular crack model. Extreme care has to be taken while choosing Δs for a proper representation of the slip weakening law. Thus, Δs is chosen as a compromise between precision and time of calculation.
For our simulation, the algorithm proposed by Kame et al. in [14]is used, above all for the calculation of the static pre-slip and the rupture along the main fault. For the consideration of the second fault, it is different because they studied a branched fault. In this algorithm , a nucleation is modelling on a main fault to enable a rupture propagation along it. Using the BIEM, the slip velocity is calculated along the fault until the fault stops slipping. The slip rate history along it, and the resulting stresses that are radiated, are used to study the rupture along the second

one. In our simulation, we assume that the eventual rupture along the second fault has no imfluence on the rupture on the main fault. Fortran 77 is used to write and run the code.

3.2.1 Static pre-slip on a nucleation zone

In order to nucleate dynamic rupture, we first assume, as in [13] and [14], a nucleation zone in a static equilibrium state on the main fault whose slip respects the slip-weakening boundary condition. Actually, we allow slip in a region of length L_{coul} sligthly larger than the minimum nucleation size L_c given by the equation 2.36 so that a dynamic rupture may begin and prevent slip outside this region. The static equilibrium is found when the slip and the stress field due to the initial stress and the slip in the nucleation region satisfies the slip-weakening law.
If the nucleation zone consists of N cells. To determine the N unknown pre-slips in a static equilibrium satisfying the slip-weakening Coulomb friction law, we have to solve the N-equations:

$$T_t^l = \sum_i D^i K_{t,static}^{l-i} + T_t^l(0) \tag{3.10}$$

using the equations 3.8 and where $T_t^l(0)$ is the shear traction on the cell l due to the initial stress. But, according to the slip-weakening law, T_t^l is a function of D^l: $f(D^l)$. We try to find the solution of:

$$\sum_i D^i K_{t,static}^{l-i} + T_t^l(0) - f(D^l), \quad \text{for } i = 1, 2, ..., N \tag{3.11}$$

The solution is numerically determined by using the Newton-Raphson method.
The distribution of slip along this initial nucleation region causes an initial stress concentration which is slightly larger than the peak strength at the both tips of the zone and so enables propagation of the rupture at the first dynamic time steps. In some of our simulations, the length of this initial region is not comparable to the theoretical minimum nucleation size. To satisfy the boundary condition and to enable the propagation of the rupture, we had to increase this length, L_{coul}. We explain this difference by, first, the comparison between the minimum nucleation size, L_c, and the static end zone, R_0: one of the most important assumptions of Palmer and Rice [18], to apply elastic linear analysis to the slip weakening law, is that R_0 has to be smaller than all the other dimensions, which is not the case of some of our simulations. Secondly, the problem of the Newton-Raphson method (solve f(x)=0) is that it does not work near a singularity or near the minimum of the function f studied. The convergence of the method depends on the first value chosen for x. The slip weakening law has a singularity at $D = D_c$.

3.2.2 Rupture along the main fault

At the time step zero, we begin the propagation. Spatio-temporal rupture evolution is determined following this procedure, for every time step, n:

Rupturing zone

We evaluate the shear and the normal tractions at each rupture tip, using the slip rate history as in the equations 3.6 and adding the static slip contribution :

$$T_t^{l,n} = \sum_{k=1}^{n}\sum_i V^{i,k} K_t^{l-i,n-k} + \sum_i D^i K_{t,static}^{l-i} + T_t^l(0) \tag{3.12}$$

$$T_n^{l,n} = \sum_{k=1}^{n}\sum_i V^{i,k} K_n^{l-i,n-k} + \sum_i D^i K_{n,static}^{l-i} + T_n^l(0) \tag{3.13}$$

By comparing the shear traction with the peak strength at the same position, we judge whether the tip extends or not.
The geometry of the fault is fixed so when the rupture reaches the end of the fault, we fix the tip at this position and the rupture cannot extend more, although continued slip can occur over the region that ruptured.

Slip velocity on the rupturing zone

We determine the slip velocity solving the coupled equations 2.29 and :

$$T_t^{l,n} = K_t^{0,0} V_{l,n} + \sum_{k=1}^{n}\sum_i V^{i,k} K_t^{l-i,n-k} + \sum_i D^i K_{t,static}^{l-i} + T_t^l(0), \tag{3.14}$$

$$T_n^{l,n} = \sum_{k=1}^{n}\sum_i V^{i,k} K_n^{l-i,n-k} + \sum_i D^i K_{n,static}^{l-i} + T_n^l(0) \tag{3.15}$$

By convenience we use the notation $T_{t,past}^{l,n}$ as the shear traction due the slip velocity history prior to time step n, the static slip and the initial stress. We remind that the normal traction is independent of the current slip velocity. Practically, we suppose that we know the slip velocity history until $(n-1)$ step. We want to determine the current slip velocity $V^{l,n}$ that satisfies the slip-weakening law and only for the cells l for which:

$$T_{t,past}^{l,n} > (-T_n^{l,n}) F(D^{l,n-1}) \tag{3.16}$$

where $F(D)$ is the slip weakening on the rupturing zone.

1. If $D^{l,n-1} > D_c$, the coupling equation gives:

$$V_{l,n} = \frac{1}{K_t^{0,0}} \left[\mu_d(-T_n^{l,n}) - T_{t,\text{past}}^{l,n}\right] \tag{3.17}$$

2. If $0 < D^{l,n-1} < D_c$, we solve algebraically the coupled equations:

$$\begin{cases} T_t^{l,n} = K_t^{0,0} V^{l,n} + T_{t,\text{past}}^{l,n} \\ T_t^{l,n} = \left[\mu_d + (\mu_s - \mu_d)(1 - \frac{D^{l,n-1}}{D_c} - \frac{V^{l,n}\Delta t}{D_c})\right](-T_n^{l,n}) \end{cases} \tag{3.18}$$

Assuming the relation 3.16, there is a unique solution which lies with $V_{l,n} > 0$, provided that:

$$\frac{\mu}{2c_s} > (-T_n^{l,n})(\mu_s - \mu_d)\frac{\Delta t}{D_c} \tag{3.19}$$

If we use the definition of the different parameters given in the section 2.2.1, the inequality assuring a unique positive slip velocity becomes:

$$\Delta s/R_0 < 8/(\sqrt{3}\pi) \quad \text{that is} \quad \Delta s/R_0 < 1.47. \tag{3.20}$$

Actually, R_0 depends on the the pre normal stress, it is then different along a curved fault. So Δs has to satisfy this condition all along the fault considered.

Thus, we determine the slip velocity on each fractured element.

Damping parameter

We introduce an artificial damping term to eliminate a short-wavelength oscillations that appear in slip velocity, due to the abrupt progresses of the fracture front along the discretized fault trace. They become evident for large numbers time steps and grow rapidly, and invalidate the results, as explained in [28], [13] and [14]. We try to eliminate the oscillations introducing an attenuation term. After calculating the slip velocity V^i over ruptured region, at each time step, n, we transform it to damped one:

$$V_{\text{damp}}^{i,n} = V^{i,n} + \alpha\left(V_{\text{damp}}^{i-1,n} + V_{\text{damp}}^{i+1,n} - 2V_{\text{damp}}^{i,n}\right) \tag{3.21}$$

The unknown $V_{\text{damp}}^{i,n}$ can then be solved numerically using a matrix inversion.
The choice of α is very delicate: stronger artificial damping suppresses not only the oscillations but the quantity of slip also. A compromise has to be done between stability and plausibility of the solution.
Comparing their numerical results using damping parameter and with an analytical solution, Yamashita and Fukuyama, [28] have shown that the value $\alpha = 1/2$ give stable and plausible results. So this value is chosen for the simulations.

Time calculation: way to reduce it

In their simulation for branched faults, to calculate the convolution represented in the equations 3.6, Kame et al., [14] calculate the kernels $K^{l-i,n-k}$ for each calculation of stress in the cell l at time step n. If N is the number of cells discretizing the fault, the time of calculation is proportional to N^5. So, more refined the discretization is, more time steps we have to consider and more longer the time calculation is. To reduce the time calculation, we have first thought about the paralelization of the code. Using the HPC clusters of the laboratory composing by 12 processors, we have tried to parallelized the code. It does not succeed because of the communication

between the processors. Indeed, to calculate the stress for example on a particular cell and at a particular time step, the slip velocity history of all the cells for prior time steps are needed. So, the different processors have to communicate their calculations, which takes a certain time. The time won in dividing the calculations by 12 is lost in communicating their memory.
Actually, as the notation $K^{l-i,n-k}$ implies, the kernel depends only on the distance between the two cells considered and on the relative time. Instead of calculating for every time step and every cell the kernels without considering the previous calculations, for straight faults where the distance between two points is just a multiple of Δs, we calculate in a first step all the kernels according to a distance and a relative time step and save it to a file. For the different convolutions, we have just to read the file and use the convenient kernel. The complexity is so divided by N.
The problem of this method is that it is immediately relevant only for straight fault and not curved one, for which the distance between two cells is particular to these two cells according to the geometry of the fault.Possibly, a look-up table with interpolations could be used for curved faults, although we have not yet studied that.

Stress distribution within the domain

Using the same parameters of implementation as mentioned earlier, we can discretized the whole domain (around the fault) giving an orientation to each cell. Thus, using the same discretized representation of stress due to the slip velocity history of the fault in equation 3.6, we can calculate the shear stress and the normal stress for each cell of the domain for each time step. These calculations give to us the dynamic stress distribution around the fault due to the rupture propagation along it. The region where a rupture is possible can be determined for every time step. Potential sites for nucleation of rupture off the main fault can thus be determined. This calculation can be parallelized because once we have calculated the slip rate history for all the fault and for each time step, we have just to communicate the $V^{l,n}$ for every l and every n to the 12 processors at the beginning so that we can ask each processor to calculate only a part of the set of time steps. There is no need for communication between processors..

3.2.3 Study of the second fault

Here, the jump of the rupture from the main fault to the second one is studied. As signaled by Harris and Day, [10], three scenarios are possible, depending on the geometrical characteristics of the faults.

1. The rupture dies at the end of the first fault segment.
2. The rupture triggers the second fault segment but then runs out of energy and stops propagating
3. The rupture triggers the second fault segment then continues propagating.

Single nucleation on the second fault

With the slip rate history of the first fault given (from our earlier calculation), we want to study the possibility for the rupture to jump. We take a time step when the rupture has not reached the ends and we calculate the tangential and normal tractions all along the second fault for each time step and we suppose that a rupture can nucleate when in one cell the tangential traction is higher than the local peak strength. If this happens we later apply the algorithm explained in the last section for the propagation of the rupture and the calculations of slip velocity in the ruptured region. For the calculations of stress we consider then the rupture on the first and on the newly activated fault. We suppose just that the rupture on the second one has no influence on the first, that means that we do not re-calculate the change of the stress on the first fault due to the rupture on the second one.This is sensible because slip on the first fault has stopped or nearly stopped by the time waves would reach it from the second fault. By the time waves from any small further slip on the first fault made their way back to the second, the rupture front would have moved much further along the second fault.

Multiple nucleations on the second fault

Depending on the geometry of the second fault, multiple nucleation sites may exist. A rupture can nucleate in different time steps and on different isolated locations. So, if a rupture has already nucleated, we continue to test along the region which has not ruptured if a nucleation is possible (Figure 3.2). If two nucleations are possible for example, we just must take care to join the tips (iL(i) and iR(j) represented in figure 3.2) of the two ruptured regions when it is possible. For the propagation of the rupture and the calculation of slip velocities for each region, the same algorithm is still used.

Figure 3.2: Multiple nucleations: the first fault has ruptured. iL and iR represent respectively the left and the right tip of each ruptured region.

Chapter 4

Simulations for various fault geometries and parameters

We have chosen to deal first with the same configuration of faults and geophysical conditions as Harris and Day [10] to test the code and to draw first conclusions about the jump of the rupture. Using the BIEM, explained in the last chapter , we study more complex faults and show the importance of the geometry to enable or prevent the jump.

The length of the main faults studied here, like in the elastostatic study, is L=28 km. The origin of the along-fault coordinate is at the center of this fault. The prestress is the same for all the cases (table 1.1). The slip weakening critical distance, D_c, defined in the section 2.2 is also fixed and taken as 10cm.

4.1 Single fault study

4.1.1 Influence of the different parameters

Harris and Day studied the influence of the stress drop $(\sigma_{xy}^0 - \tau_r)$, the S ratio and the pre stress on the phenomenon. They chose four different mechanical conditions, summarized in the table 1.1.

Because the value of μ_s and μ_d are different according to the cases, the parameters defined in the chapter 2 (the energy release rate, the dimension of the slip weakening zone, the minimum nucleation size) are different. As explained in the last chapter, we have to choose the cell size, Δs, for the discretization of the fault and so the time step Δt, according to the dimension of the initial slip weakening zone R_0. The smaller is Δs the better is the representation of the slip weakening zone but longer is the time of calculation. Thus, the simulation parameters are chosen as a compromise between the precision and the time of calculation. The table 4.1 gives the value of the parameters for each case.

	Case A	Case B	Case C	Case D
R_0 (km)	0.236	0.786	0.133	0.442
$R_0/\Delta s$	2	3	1	3
$L/\Delta s$	120	55	110	95
$L_{nucl}/\Delta s$	4	6	8	18
Δt (s)	0.01	0.02	0.01	0.01

Table 4.1: Simulation parameters for each case: R_0 is explained in the section 2.2, L_{nucl} is the nucleation length ($L_{nucl} > L_c$) and L is the length of the fault. See table 1.1 for further details.

Rupture velocity

From an initial crack with a critical length, the rupture propagates along the straight fault. It is interesting first to point out the time for the rupture to reach the end of the fault for each case (table 4.2). As explained in the section 2.2.2, the

	Case A	Case B	Case C	Case D
T (s)	2.30	2.66	4.65	5.0

Table 4.2: Time taken to reach the end of the faults

rupture velocity depends on the S ratio. The smaller it is, the higher is the rupture velocity and so the smaller is the time taken to reach the end. Figure 4.1 represents the rupture tip propagation velocity, v_r (as v_r/c_s where c_s is the shear wave speed). Since space and time are discretized in uniform steps, the possible rupture velocities reported by our procedures are quantified to a finite set, hence the appearance of the plots. Only averages over multiple fluctuations are meaningful. Using the figure 9 of [2] for example, according to the length of the fault, as specified in [10] the rupture velocity is subshear when $S > 1.63$ and supershear when $S < 1.63$. For the case A ($S = 0.49$), v_r tends to approximately $1.7 - 1.8c_s$ and for the case D ($S = 1.65$) $0.8c_s$. $c_p/c_s = \sqrt{(}3) = 1.73$ so remarkably this is moving a little faster, on average, than c_p; this is probably an effect of the small$R_0/\Delta s$.
Even if supershear rupture velocity has not been observed a lot in nature (an example is the 1999 Izmit turkey earthquake), they are important to study and easier too because the time of calculation is a lot of smaller: the time needed for the faults to finish to slip is smaller.
The table 4.2 shows that the stress drop has a certain influence on the time to reach the end or more generally on the rupture velocity, comparing the case A (stress drop $[\sigma^0_{xy} - \tau_r]/(-\sigma^0_{yy})$ of 0.3) and the case B (stress drop of 0.09) for example.

Figure 4.1: Rupture velocity V_r for a case where it is super-shear (case A, at the left) and another one where it is sub-shear (case D). The quantities are non dimensional.

Slip and slip velocity along the fault

Figure 4.2 shows the evolution of the displacement discontinuity or slip δ (as $2\mu\delta/[-\sigma_{yy}^0 R_0]$) all along the fault (in terms of cells as $x/\Delta s$, Δs is specified for each case in table 4.1) for each case. The distribution of slip for each case is symmetrical

Figure 4.2: Slips for the four cases whose conditions are in the table 1.1, all along the faults, for each 0.4s (that is $N\Delta s/2c_p$ where Δs is specified in the table 1.1 for each case and N is equal to 20 for the case B and 40 for the other cases) . The quantities are non dimensional. Kinks in the cases A and B correspond to transition to supershear rupture

because the nucleation of the rupture is simulated symmetrically around the center and because the mechanical conditions are homogeneous along the fault.

The shape of the slip for a fixed time is approximately the same: parabola with a maximum at the center of the fault. But for a super shear rupture velocity, some parabolas present kinks, this is due to the transition of super shear rupture velocity. Besides, calculating the slip δ for each case from the non dimensional values, we can underline the influence of the stress drop. Indeed, for example, for the cases A and C where the nondimensional stress drop is 0.3, the maximum slip is between 8 and 9m whereas for the cases B and D, it is between 2 and 3 m. From our calculations we can plot also the slip velocity all along the fault for each time step and make movies of their variation. For each case, the movies are available at my web site and the link is:

http:\\esag.harvard.edu\fliss\A_slipvel.avi
http:\\esag.harvard.edu\fliss\B_slipvel.avi
http:\\esag.harvard.edu\fliss\C_slipvel.avi
http:\\esag.harvard.edu\fliss\D_slipvel.avi

These calculations give equivalent observations. the larger is the stress drop the larger is the slip velocity. The features according to the S ratio and so the rupture velocity are slightly different with a presence of several peaks when the rupture velocity is supershear.

4.1.2 Slip velocity

Here is shown the slip velocity for a super shear rupture velocity only: the case A. The figure 4.3 is composed from some framess of the movie available at the link:

Figure 4.3: Slip velocity v (as $V* = \mu v/(-\sigma_{yy}^0 c_p)$ vs. $x* = 2x/R_0$) in the case A for several time steps, $N = 4c_p t/R_0$.

http:\\esag.harvard.edu\fliss\A_slipvel.avi
and represents the slip velocity v, as $V* = \mu v/(-\sigma_{yy}^0 c_p)$ vs. the cell $x* = 2x/R_0$ for several time steps $N = 4c_p t/R_0$. The larger peaks represent the tips of the ruptured zone in the fault. The smaller ones, for example at $N = 80$ until $N = 240$, are due, we think, to the propagation of elastic waves. After the rupture reaches the end, the feature is less obvious: some peaks propagate from the end to the center of the fault. At $N = 500$, we can observe that at its ends the fault begins to lock (or has a very low slip velocity) and the regions where the slip velocity tends to zero increase from the end to the center. After $N = 700$, the variation of the slip velocity may be less justifiable physically. Indeed, some regions locked in previous times slip again as we can observe at $N = 700$ and $N = 850$. The slip weakening law incorporated in our study as the rupture criterion does not account for the re-strengthening of the fault when it has stopped slipping. By $N = 700$ all the cells have slipped beyond D_c requiring them to overcome just τ_r to slip further.

4.1.3 Stress Distribution

Once the slip rate history of the fault is known, we can calculate the stress distribu-

□

Figure 4.4: Stress distribution around the first fault for case A, for elements parallel to the x-axis. Only the region of potential failure is shown, i.e. regions where the contoured quantity $\sigma_{xy}/(-\mu_s \sigma_{yy}) > 1$. Representation for several time step: $N = 2c_p t/R_0$

tion (tangential and normal tractions) for each cell and each time step. We have to specify an orientation to a prospective fault: they are considered to be as parallel to the first fault. The normal $\sigma_{yy}/(-\sigma_{yy}^0)$ and the tangential $\sigma_{xy}/(-\sigma_{yy}^0)$ tractions are calculated. Figure 4.4 represents contoured quantity $\sigma_{xy}/(-\mu_s \sigma_{yy})$: only the regions of potential failure defined by $\sigma_{xy}/(-\mu_s \sigma_{yy}) > 1$ are shown for several time steps: $N = 2c_p t/R_0$. Only the region $x > 0$ is represented, because of the symmetry of the problem we can imagine the region $x < 0$. For this phase of the work, because of the time of calculations, despite the parallelized code used, to descretize the fault and the domain the cell size here Δs is chosen equal to R_0. A movie is available at:
http:\\esag.harvard.edu\fliss\Tcoul.avi
and represents the same quantity for every 10 time steps.
The first observations are the same as in the static study: differences between compressional and dilational sides in terms of shape, of maximum "jumpable" distance,...

We observe here a very important result: until the rupture reaches the end of the fault, the tangential traction induced by the initial stress and the distribution of slip is not enough to enable a rupture beyond the fault. After it reaches the end, a release of energy, explained by Freund and Fossum in [8], enables a stress distribution around the fault whose characteristics make a rupture possible in some second parallel faults. The shape of the regions where a nucleation is possible is approximately the same than the one found in the static study. But its dimensions change dynamically.
From the time step $N = 170$ to $N = 280$, a region where the nucleation is possible seems to move away from the initial shape on the compressional side. It seems to correspond to the smaller peaks appearing in the distribution of the slip velocity which propagates slower than the rupture.
The deformation of the shape with the time can explain some observations of Harris and Day in [10]: the rupture nucleates on one fault, in a particular case, but does not propagate along the fault. The nucleation in the fault does not favor the propagation and the stress due to the displacement discontinuities along the first fault is not enough to enable the rupture propagate.
Finally, the representations of the stress distribution for the first time steps (when the rupture is still propagating along the first fault) which are not represented in the figure 4.4 shows a small region with high shear stress propagating with the rupture and present especially on the dilational side of the fault. We can establish a link with [19] and [23] (the crack model are however different) and their observations of the secondary faulting in the dilational side for a right lateral slip. In our model, this region around the rupture can explain the tendency of off-fault secondary failure.

Final shear stress along the fault

Figure 4.5: Final shear stress $\sigma_{xy}/(-\sigma^0_{yy})$ in the case A. The residual stress $\tau_r/(-\sigma^0_{yy}) = 0.3$ in this case. The nucleation on the first fault is simulated around the middle.

From the last calculations, we can plot the shear stress $\sigma_{xy}/(-\sigma^0_{yy})$ along the fault once it stops slipping: figure 4.5. Theoretically, the slip weakening law imposes to the final tangential traction to be equal to the residual strength, which is in this case $\tau_r/(-\sigma^0_{yy}) = 0.3$. But the figure shows that the final stress is below this value all along the fault: we have dynamic overshoot.

Rupture nodes

□

Figure 4.6: Map view of potential nucleation sites and associated time step ($N = 2c_pt/R_0$) for the case A. The main fault is shown by thick black line. Other possible faults are shown by thin lines. The distances are in km, $R_0 = 0.236$km.

Here we suppose that there is a second fault at a fixed distance from the first one and we want to know if a nucleation is possible and if it is, where and when a nucleation occurs first. From the stress distribution calculations, at a fixed y, we determine first if a x exists such that the tangential traction overcomes the peak strength. If it does, we determine the time and the location of the first nucleation. The results are given in the figure 4.6. Only a part of the main fault is represented and for each y (km) the potential nucleation sites (x and y in km) are represented with black circles and the associated time step N with green squares ($N = 2c_pt/R_0$). We find some observations signalled in the static study and just above, especially the differences between the compressional and the dilational sides emphasized already in [10]. For example, the nucleation sites on the compressional side are far away from the tip of the fault; on the contrary, the ones on the dilational side are just below the fault. The supplementary information induced by this plot is the difference in terms of time: the compressional steps are jumped more quickly than the corresponding dilational.

This single fault study is not enough for the study of the jump. Indeed, it gives information about when and where the static friction level is exceeded but not whether the stress concentration is enough to favor a propagation and how much time is necessary for the propagation. That is why the study of different second faults reveals to be primordial.

4.1.4 Comparison with a nucleation which occurs near the end of the fault

All the nucleations in the cases above are simulated at the center but it could be interesting to see what is the influence of the location of the nucleation. That is why the same calculations are done in the case where the cell size is chosen equal to R_0 (even if it is not really precise, to have rapid results, a larger cell size had to be chosen). The nucleation is simulated 40 cells far from the center in the negative x diirection, that is about 9.5km.

Slip and Slip velocity

Figure 4.7 represents the slip δ (as $D* = \mu\delta/(-\sigma^0_{yy}R_0)$) along the fault (as $x* = x/R_0$) for each 0.4s. Figure 4.8 represents the slip velocity v (as $V* = \mu v/(-\sigma^0_{yy}c_p)$

Figure 4.7: Slip δ (as $D* = \mu\delta/(-\sigma^0_{yy}R_0)$ vs. $x* = x/R_0$) in the case A for each 0.4s (as $10R_0/c_p$) when the nucleation is simulated around $x* = -40$ from the center ($x = 9.5km$).

Figure 4.8: Slip velocity v (as $V* = \mu v/(-\sigma^0_{yy}c_p)$ vs. $x* = x/R_0$) in the case A if the nucleation is simulated around $x* = -40$, for several time steps, $N = 2c_pt/R_0$.

along the fault (as $x* = x/R_0$) for several time steps $N = 2c_pt/R_0$. The slight oscillations are due to the parameters of the discretization chosen.
The average and maximal slips are quite the same as for a nucleation at the center. The maximum is still at the center of the fault.
The variation of the slip velocity is available at:
http:\\esag.harvard.edu\fliss\A_slipvel_nucl-40.avi

Using the same crack model, Johnson in [12] claims the appearance of a self healing pulse, a phenomenon which occurs after the rupture reaches one end and continues propagating unilaterally. His results show that after the rupture stops at one side, the fault seems to lock in this place, the region where the fault is locked increases and the tip of this region propagates at velocity approaching the rupture velocity. In our study, the cause of lack of comparably completehealing is unknown. That can be due to limited resolution in this case or to other procedure related to our algorithm or to the fact that the rupture speed is super shear here. Actually, we do not know when Johnson considers that the fault is locked; if it is when the slip velocity is precisely equal to zero or if it is lower than a small value.
Besides, obviously all the quantities (slip, slip velocity, stress distributions,...) are not symmetrical about the center of the faults.

Final shear stress

Figure 4.9: Final shear stress $\sigma_{xy}/(-\sigma^0_{yy})$ in the case A. The residual strngth $\tau_r/(-\sigma^0_{yy}) = 0.3$. The nucleation on the first fault is simulated around $x* = -40$.

The figure 4.9 plots the shear stress $\sigma_{xy}/(-\sigma^0_{yy})$ along the fault well after the fault stops slipping. Like for a nucleation at the center, the final shear stress is globally lower than the residual strength $\tau_r/(-\sigma^0_{yy})$(=0.3), except near the location of the nucleation, at the left end of the fault. There is again dynamic overshoot except very near the left end.

4.2 Second fault: parallel. Case of a super-shear rupture velocity

In this section, we study the jump of the rupture from the first fault, whose length is $L = 28km$ to a second parallel one whose length is L'. We can change what we call the overlap and the width (figure 4.10) according to the cases. The case A, for

Figure 4.10: Geometry of the faults and some useful notations.

which the rupture velocity is super shear, is considered here.

4.2.1 A case of a jump of the rupture

First we wanted to observe a jump with our model and calculations. Using the Harris and Day results in [10], the static study and the first results about a single fault, we have chosen to study a second fault with the same length than the first one ($L' = 28km$ that is $L_c/R_0 \sim 120$), an overlap of approximately 5 km (that is $22R_0$) and a width of approximately 1 km (that is $5R_0$). Taking to account the time of calculation, Δs is fixed equal to R_0. Thus, the time step is equal to $N = 2c_p t/R_0$ and the origin of time is again the beginning of the rupture on the first fault.
The dynamic calculations give a nucleation site at 2 cells (that is $2R_0$) before the end of the first fault. It occurs at the time step $N = 145$ (that is 2.86 s). These results coincide with those given in figure 4.6. Besides, the rupture propagates on the second fault and begins at the time step $N = 149$ (that is 2.94s) that is quite instantaneously.

Slip

Figure 4.11: In the case A, slip δ (as $D* = \mu\delta/(-\sigma^0_{yy}R_0)$ vs. $x* = x/R_0$) for each 0.4 s ((that is $10R_0/c_p$), along a second fault, parallel: length = 28 km ($120R_0$), overlap $\sim$ 5 km ($22R_0$) and the width$\sim$ 1km ($5R_0$).

The figure 4.11 represents the slip δ (as $D* = \mu\delta/(-\sigma^0_{yy}R_0)$) all along the fault (in terms of cell $x* = x/R_0$) for each 0.4s. The maximum slip is higher than in the first one: about 11m against about 8.5m for the first fault.

Slip velocity

The figure 4.12 represents the slip velocity v (as $v* = \mu v/(-\sigma^0_{yy})R_0$) all along the fault (in terms of cell $x* = x/R_0$) for several time steps $N = 2c_p t/R_0$. The

Figure 4.12: In the case A, slip velocity v (as $V* = \mu v/(-\sigma^0_{yy} c_p)$ vs. $x* = x/R_0$) for several time steps, $N = 2c_p t/R_0$, along a second fault, parallel: length 28 km ($120R_0$), overlap 5.1 km ($22R_0$) and the width 1.2km ($5R_0$).

oscillations are due to the choice of the cell size. The variation of the slip velocity is available at:

http:\\esag.harvard.edu\fliss\A_slipvel_o5h1.avi

The nucleation occurs near the end of the second fault but the rupture continue to propagate all along the fault and reaches the other end because of the stress created first by the displacement discontinuities along the first fault and secondly by the slip along the second one. The feature of the slip velocity is slightly the same than the one found for a single fault with a rupture nucleation near the left end. But once the rupture reaches the end the feature is quite different because of the interactions between the two faults.

4.2.2 Impact of the distance between the two faults

According to [10] and the figure 4.6, if we increase the width until a certain value a nucleation is still possible in different location but above all after a longer time. Here, a larger width is chosen for which a nucleation is sure (according to our calculations): $\sim$ 3km ($15R_0$).

The dynamic calculations give a nucleation site at 4 cells (that is $4R_0$) before the end of the first fault. It occurs at the time step $N = 298$ (that is 5.88 s). These results coincide with those given by the figure 4.6. Besides, the rupture propagates and begins at the time step $N = 407$ (that is 8.03s). Thus, there is a delay between the nucleation and the propagation of the rupture. The larger is the width, the longer is the time needed for the rupture to jump and after to propagate.

4.2.3 Impact of the length of the second fault

Figure 4.13: In the case A, slip δ ($D* = \mu\delta/(-\sigma^0_{yy} R_0)$ vs. $x* = x/R_0$) for each 0.4 s(that is $10R_0/c_p$), along a second longer fault, parallel: length 37.3 km ($160R_0$), overlap 25.7 km ($110R_0$) and the width 1.2km ($5R_0$).Two cases simulated: the nucleation on the first fault occurs around its center (at the left) and around the cell $x* = -40$ (at the right)

We know with the previous results that the rupture propagates all along the fault when the overlap is not very long. The stress created by the first fault favor

the nucleation and the bilateral propagation when a short part of the second fault is below the right half of the first one. Now, we increase the overlap and make a part of the second fault below the left half of the first one. Because of the symmetry of the problem we can understand that this region has been shielded from stress and does not favor the slip and the propagation of the rupture along this part of the second fault.
The figure 4.13 represents the slip δ (as $D* = \mu\delta/(-\sigma_{yy}^0 R_0)$) for each 0.4s, all along a long fault (in terms of cell $x* = x/R_0$) whose length is about 37.3 km ($160R_0$), the overlap is about 25.7 km ($110R_0$) and the width is again about 1.2 km ($5R_0$). The location of the nucleation on the first fault can have an influence: the slip is represented for a nucleation on the first fault at the center (left plot) or near the end (right plot).
It is obvious that the rupture propagation is slowed down because of this geometric configuration, the tips of the ruptured zone does not propagate in the same way, the slip distribution is not symmetrical at all, the maximum slip does not correspond to the center of the fault,... However, the rupture velocity is so large at the beginning that even if it is slowed down it reaches the left end.
The feature of the slips in the two cases of nucleation seems to be the same. But when the nucleation occurs near the end of the first fault, the rupture velocity seems to be higher than the one in the other case. Besides, in this case, obviously, the jump occurs 4.31s (219 time steps) after the beginning of the simulation against 2.86s (145 time steps) for the other case. The rupture reaches the end more quickly than when the nucleation occurs at the center (9.27s (470 time steps) against 10.75s (545 time steps)) and the slip velocity is higher. The movies of the slip velocity are available at:

http:\\esag.harvard.edu\fliss\A_slipvel_long_nucl0.avi
http:\\esag.harvard.edu\fliss\A_slipvel_long_nucl-40.avi

Thus, these first examples provide some informations about the influence of the geometry on the interactions between two faults and so on the jump of the rupture from one fault to the another one, its nucleation and propagation.

4.3 Second fault: parallel. Case of a sub-shear rupture velocity

The geometry of the faults and the notations are the same than those given in the figure 4.10. Here, the conditions of the case D (see the tables 1.1 and 4.1) for which the rupture is sub shear (see figure 4.1) are considered. The cell size and the time step is different for this case. Because of the values of the different parameters we have chosen: $\Delta s = R_0/3$ and so $\Delta t = R_0/6c_p$

4.3.1 The rupture can jump smaller distances

As shown in [10] and in the static study, with this S ratio, the rupture can jump smaller distance, with a maximum of 1 km given in [10] and of 0.7 km in the dilational side given by the static study. We have chosen first a second fault whose length is the same than the first one, 28 km (190 cells) the overlap is about 5 km (35 cells) and a width of 500m (3 cells), for which we are sure that a rupture nucleates. The nucleation on the first fault occurs at its center.
The rupture nucleates in one cell before the end of the first fault 4.82s after the beginning (392 time steps) and propagates instantaneously.
Thus the change of the conditions and the S ratio especially makes the maximum "jumpable"distance smaller, the rupture velocity slower and so the time of propagation and the time needed to jump longer.
The value of the slip and the slip velocity, the last one available at:

http:\\esag.harvard.edu\fliss\D_slipvel_o5h500.avi

are larger along the second fault than the first one: for example the maximum slip is about 3.5 m on the second fault against less than 3m on the first one.
The rupture propagates bilaterally and until the ends of the fault.

4.3.2 Impact of the length of the second fault

We have seen in a previous simulation that when the second fault is very long and below the first one, the rupture is not encouraged and its velocity decreases dramatically. In the case of a super shear rupture velocity, the rupture reaches nonetheless the ends of the fault. Here, the case of a long second fault under conditions for which the rupture velocity is sub shear is treated.

Slip

Figure 4.14: In the case D, slip δ (as $D* = 3\mu\delta/(-\sigma^0_{yy}R_0)$ vs. $x* = 3x/R_0$) for each 0.4 s ($20R_0/3c_p$), along a second longer fault, parallel: length 35.4 km ($240R_0$), overlap 25.8 km ($175R_0$) and the width 0.44km ($5R_0$).Two cases simulated: the nucleation on the first fault occurs around its center (at the left) and around the cell $x* = -60$ (at the right)

The figure 4.14 represents the slip δ (as its non dimensional value $D* = 3\mu\delta/(-\sigma^0_{yy}R_0)$) all along the fault (for each cell $x* = 3x/R_0$) and for each 0.4s (that is $NR_0/6c_p$). Two cases of nucleation on the first fault are again studied: at its center and near the end (around 9 km far from the center (cell -60)), the plots are respectively at the left and the right in the figure 4.14.
Thus, for the two cases, the rupture along the fault is slowed down because of the motion along the first one and finally stops before the end.
We observe again that the propagation on the second fault is more favored when

the rupture on the first fault initiates near its end.

Slip velocity for nucleation at -60

Figure 4.15: In the case D, slip velocity v (as $V* = \mu v/(-\sigma_{yy}^0 c_p)$ vs. $x* = 3x/R_0$) for several time steps $N = 6c_p t/R_0$, along a second longer fault, parallel: length 35.4 km ($240R_0$), overlap 25.8 km ($175R_0$) and the width 0.44km ($5R_0$). Here, the nucleation on the first fault is simulated around the cell $x* = -60$.

The figure 4.15 represents the slip velocity v (as $V* = \mu v/(-\sigma_{yy}^0 c_p)$) all along the fault (as $x* = x/R_0$) and for several time steps ($N = 6c_p t/R_0$) when the nucleation on the first fault occurs near its end. The movie is available at:
http:\\esag.harvard.edu\fliss\D_slipvel_long_nucl-60.avi

This is just to show the difference between the propagation of the tips of the ruptured zone along the second fault and how the slip velocity decreases during the propagation to the left.

4.4 More and more complex geometry for the second fault. Case of a super-shear rupture velocity

Figure 4.16: Geometry of the two faults with the notations used. d is the distance between the centers of the two faults O and O', ω is the angle the second fault makes with the x-axis.

The relevance of the BIEM is the possibility of discretizing easily faults with complex geometry. In this section, the jump of the rupture is studied between the same main fault as the last sections (straight with a length of 28 km) where the initiation of the rupture is modelled at its center and a second more complicated fault whose geometry will be explained for each case. The rupture velocity is super shear and the mechanical conditions used are those of the case A (1.1)
For straight faults parallel to the x-axis, the value of the ratio $\sigma_{xx}^0/\sigma_{yy}^0$ does not matter because it does not intervene in the normal traction neither in the tangential one. That is why Harris and Day [10] do not specify this value for their fault. However for other faults, it has to be fixed. As explained in the static study in the part 1, we have chosen a ratio $\sigma_{xx}^0/\sigma_{yy}^0 = 1.5$ and all the dynamic calculations are done with this value.
This section will provide some conclusions about the influence of the geometry of the faults on the rupture.

4.4.1 Case of a straight oblique fault

First, we consider second faults which are straight but oblique, that is, they makes an angle ω with the x-axis (figure 4.16), are considered. The rupture for several configurations of faults (with different values for d, ω) is simulated.

Results

First, $d = 14km(60R_0)$ and the width is equal to 1km ($5R_0$). For $\omega = 15^o$ and $\omega = 30^o$ there is no nucleation along the second fault.
If we increase d, there is also no nucleation.
As a matter of fact, for faults with positive orientations, because of the characteristics of the pre stress field, the normal stress to this fault is too high and so the peak strength too. A more detailed analysis of the calculations shows that the stress created by the displacement discontinuities along the first fault which makes the tangential stress increased is not enough for the fault to rupture in terms of the slip weakening coulomb friction law.
Here, we emphasize the importance of the pre stress orientation on the rupture along specific fault geometry.

4.4.2 Simplification of a curved fault

Figure 4.17: Geometry of the two faults in the xy plane, $x* = 2x/R_0^o$, $y* = 2y/R_0^o$. The left half of the second fault makes an angle $\omega = 30^o$ with the x-axis. The distance between the two centers is about 11.7 km (that is 50 R_0). The kink in the second fault is at its center.

The other step for the modelling of more and more complicated second fault geometry is a second fault with two part: one part (the right one) which is parallel to the x axis and another part (the left one) which makes an angle ω with the x axis. The figure 4.17 represents the case of $\omega = 30^o$. Actually, we have simulated two cases $\omega = 15^o$ and $\omega = 30^o$, the distance between the two centers is about 11.7 km and the width is about 1km.

Precision about the simulations

As explained in the equation 3.20, there is a condition for the choice of Δs so that the algorithm works:

$$\Delta s/R_0 < 8/(\sqrt{3}\pi). \qquad (4.1)$$

This condition has to be respected all along the faults. Seeing that R_0 along an element of the fault defined in the equation 2.37 depends on the normal stress applied on this element and so depends on its orientation ($R_0 = R_0(\theta)$ when θ is the orientation of the element), in this particular case : 0 or ω. Until the end we will call R_0^o the value of the the slip weakening zone calculated for a fault parallel to the x-axis before the rupture. We have another important condition for the choice of Δs. For $\omega = 15^o$, $\Delta s = R_0$ respects all the conditions but for $\omega = 30^o$, Δs has to be smaller. $\Delta s = R_0/2$ is fine for the two orientations.

Besides, the study of oblique faults shows that the nucleation cannot occur along such oriented fault. We are sure that the rupture will occur on the parallel part in a single location and we want to see if it can propagate on the oblique part despite the stress orientations. Thus, the program which tests the possibility of multiple nucleations along the second fault (explained in the section 3.2.3) is not yet applied. A single nucleation is supposed.

Results

Figure 4.18: Position of the left and right ends of the ruptured zone (as $s* = s/R_0^o$ where s is the curvilinear coordinate, the origin is the center of the fault.) for each time step (as $N = 2c_p t/R_0^o$. The two lines at each edge indicate the length of the ruptured zone.The half length of the fault is 14km that is about 60 R_0^o.

For $\omega = 15^o$, the rupture stops abruptly for a few moments where the change of the orientations begins, and starts propagating along the oblique part more slowly than along the parallel part. Then it reaches the left end after reaching the right end. The figure 4.18 represents the bilateral propagation of the rupture and the difference between the two sides.
It appears clearly that the orientation of the left half of the fault does not favor the rupture propagation along it. This is the rupture along the parallel part which makes the propagation on the other part possible.

Figure 4.19: In the case A, slip δ ($D* = 2\mu\delta/(-\sigma_{yy}^0 R_0^o)$ vs. $s* = 2s/R_0^o$ where s is the curvilinear coordinate) for each 0.2 s(that is $10R_0/c_p$), along a second fault, whose left half is oblique and makes an angle of 30^o with the x-axis. At the left is the distribution of slip all along the fault (from -120Δs to 120Δs) and at the right is the zoom of the left one.

The larger is the orientation of the left half, the more difficult is the rupture along it, because of the pre stress field.
Indeed, for $\omega = 30^o$, the rupture stops on the oblique part, as it is represented in the figure 4.19 which represents the slip δ (as $D* = 2\mu\delta/(-\sigma_{yy}^0 R_0^o)$) all along the fault (as $x* = 2x/R_0^o$) for each 0.2s. The rupture stops at the center of the fault,

where the orientation of the fault changes abruptly, restart propagating very slowly thanks to the slip along the parallel part of the second fault, and stops again.
Thus, these abrupt changes of the orientation of the fault do not favor the propagation of the rupture because of the pre stress field: the rupture tends to slow down even to stop.

4.4.3 Case of a curved fault

The problem of the last modelling is the singularity introduced at the center when the orientation of the fault changes abruptly from ω to 0^o. To remove this singularity, the orientation has to change more progressively. This is what we have done for the last case.

Geometry

As represented in the figure 4.20, the right half of the second fault keeps parallel to the first one, the width is taken again at 1km, the distance between the two centers is still about 11.7km ($50R_0$), the width is still 1 km ($5R_0$) the orientation of the fault varies progressively from 0^o to reach 28^o at the left end of the fault.

Figure 4.20: Geometry of the two faults in the xy plane, $x* = 2x/R_0^o$, $y* = 2y/R_0^o$. The orientation of the left half of the second fault decreases from 0^o to 28^o

Precision about the simulation

For the choice of the cell size, we have seen in the last section the importance of the condition 4.1. It has to be right all along the fault and for each orientation. If we choose $\Delta s = R_0^o/2$, the condition is respected all along the fault: if $R_0(\theta)$ is the slip weakening size when the orientation is θ and Δs is such that for all θ between 0^o and 28^o: $\Delta s \leq R_0(\theta) \leq 2\Delta s$.
Moreover, here, the orientation varies progressively and with this more complex geometry, we are not sure that the rupture will initiate in a single location, the condition for the rupture can be verified in several locations along the fault because of this curvature.
That is why, the program which studies the case of multiple nucleation is applied here.

Results

Figure 4.21: In the case A, slip δ (as $D* = 2\mu\delta/(-\sigma_{yy}^0 R_0^o)$ vs. $s* = 2s/R_0^o$ where s is the curvilinear coordinate) for each 0.2 s (that is $10R_0/c_p$), along a second fault, with a curved left half

Figure 4.22: In the case A, slip velocity v (as $V* = \mu v/(-\sigma_{yy}^0 c_p)$ vs. $x* = 2x/R_0^o$ and $y* = 2y/R_0^o$) for several time steps $N = 4c_p t/R_0^o$,along the two faults , with a curved second fault

First, according to our model, the nucleation of the rupture along the second fault occurs in a single location, in the straight part of the fault and in the same location and same time as for a parallel fault (section 4.2.1). The curved part of the second fault is not involved in the nucleation of the rupture.
Figure 4.21 represents the slip δ (as $D* = 2\mu\delta/(-\sigma_{yy}^0 R_0^o))$ all along the fault (in terms of cell $x* = 2x/R_0^o$ for each 0.2s. The figure 4.22 represents the slip velocity v (as $V* = \mu v/(-\sigma_{yy}^0 c_p))$ all along the two faults (in terms of their coordinates in a plan whose origin is the center of the first fault, x axis is its direction and y axis is its normal) for several time steps N ($N = 4c_p t/R_0^o$).The variation of the slip velocity with time is available at:

http:\\esag.harvard.edu\fliss\A_slipvel3D_curv.avi

The 3d plot puts on light the jump of the rupture from the main straight fault to the second one once it reaches the end of the first one. The slowing down of the rupture when it reaches the curved part of the fault is clear but it does not stop and arrives until the end of the fault. The rupture is super shear in one direction and

sub shear in another. Obviously, the end of the straight part is reached before the one of the curved part ($N = 250$ (2.5s) and $N = 700$ (6.9s) respectively after the rupture initiates on the second fault). Moreover the values of slip and slip velocity are smaller in this curved part.
Thus, the curvature, here, enables the rupture propagation because the normal stress along the fault is increasing only progressively and not abruptly like in the last section. However it makes it quite slow.

Thus, according to our model, the jump of the rupture is possible between two faults. However, the geometry of the faults has a principal role in this phenomenon, it can enable or prevent the jump and the propagation. Moreover, the pre stress and the characteristics of the material are involved too.

Chapter 5

The jumping mechanism during the 1992 Landers earthquake

During the 1992 Landers earthquake, the rupture nucleated on Johnson Valley fault well to the south east before the branching with the Kickapoo fault, propagated on that first fault and branched to the dilational side onto Kickapoo fault, with the angle $\omega = -30^o$. It continued too along the main fault for a few kilometers. It crossed to the Homestead Valley fault via the Kickapoo fault and progressed not just forward onto the northern stretch but also backward, i.e SSE. Figure 2 represents the traces at the surface of the Earth of this rupture.
Li et al. in [16] used studies of fault zone trapped waves to show that there was transmission in a channel along the Southern Johnson Valley and Kickapoo faults and in another channel along the Homestead Valley fault, but no communication between those channels. Those results suggest that Kickapoo and Homestead Valley fault do not join, and this is for at least a depth of 10km. Felzer and Beroza [7] also show from precise relative locations of aftershocks that the fault strands remain distinct to finite depth.
We are going to apply our study to these faults to see if it supports the conclusion of a jump of the rupture. We could explain thus the backward branching observed in this case.

5.1 Choice of the parameters

For convenience, we choose the x axis parallel to the Kickapoo fault ($y = 0$) (which runs south to north) and the y axis towards the west.
Like in [14] where the branching during this earthquake is studied, μ_s is taken as 0.6 and μ_d as 0.12.
The realistic values of $J = 1MJ/m^2$ for the crack energy release rate and of $\sigma_{yy}^0 = -50MPa$ are chosen. The shear modulus $\mu = 30GPa$ is taken and the Poisson ratio is still $\nu = 0.25$.
Besides, as explained in the static study, section 1.3, there is an approximative angle

of 30^o between the most compressive stress direction and the Kickapoo fault. To define completely the pre stress field, we have to specify another value, for example the ratio, $\sigma^0_{xy}/(-\sigma^0_{yy})$. There is no rigorous way to specify this ratio, we have to choose it according to some assumptions. The case of the super shear rupture velocity is not often observed in nature; that is why we want to choose parameters so that it is sub shear. Andrews in [2] shows the influence of the S ratio on the rupture velocity. The smaller it is the larger is the rupture velocity. But the static study concludes that the smaller it is the higher is the maximum distance jumpable. Using the figure 9 of [2] (which qualify the rupture velocity according the S ratio and the ratio between the total length and the minimum nucleation size) and the static study, section 1.3, we have chosen $S = 1.3$ for which the rupture velocity is sub shear in our configuration. It leads to $\sigma^0_{xy}/(-\sigma^0_{yy}) = 0.33$ and $\sigma^0_{xx}/\sigma^0_{yy} = 1.38$.

Using this values and the section 2.2.1, we find along the Kickapoo fault (because these values depends on the orientation of the fault considered) that the slip weakening critical distance $D^o_c = 8.3$cm, the size of the low-speed slip weakening zone $R^o_0 = 122$m. Actually, the size of the slip weakening zone along all the faults varies from about 90m to 122m. Taking care of the precision and the time of the calculations, but of the condition given by the equation 4.1 too, we choose a space step $\Delta s = 40m$. Thus, all along the faults R_0 is between $2\Delta s$ and $3\Delta s$. The time step is : $\Delta t = 3R^o_0/2c_p = 0.0033s$. All the calculations are done for the first 1800 time steps (6s).

5.2 Modelling of the faults

Figure 5.1: Geometry of faults in the xy plane, $x* = 3x/R^o_0$, $y* = 3y/R^o_0$. The x-axis corresponds to the orientation of Kickapoo fault modelled straight.The orientation of Johnson Valley fault decreases from 0^o to 26^o, The orientation of left half of Homestead Valley fault decreases from 0^o to 30^o.

[25] gives a precise description of the faults involved in the 1992 Landers earthquake. With the map of this article, reproduced in the figure 2, we have done some assumptions to model the system. First by convenience we choose the x axis parallel to the Kickapoo fault.

The rupture branching between Johnson Valley and Kickapoo faults is studied in [14] where it is shown the negative angle between the two faults favors the branching and the continuation along the main fault. So we do not consider the branch and suppose just that Johnson Valley and Kickapoo are a single fault. Because the 2-D model is not sensible for crack lengths greater than the seismogenic thickness of the crust, we have to reduce the length of Johnson Valley fault as 10km, but we keep the Kickapoo fault actual length about 5km. The angle between the two faults is about 30^o. Using these data and assumptions, the origin of the system is taken at the beginning of the straight part of Johnson Valley-kickapoo system. Parallel to the x-axis, the straight fault ($y = 0$), a part of Kickapoo fault in$s > 0$ (if s is the curvilinear coordinate from the origin) measures 4 km ($100\Delta s$). In the $s < 0$, the fault begins to curve progressively along 2km ($50\Delta s$) and keeps the same orientation as 26^o along 9km ($225\Delta s$). This single fault is discretized by 100 cells at the right and 275 cells at the left of the fault.

For the modelling of the Homestead Valley fault, we know that the stepover with Kickapoo fault is about few hundreds meters (between 200 and 300m). From [25], the backward propagation seems to stop at about 4km from the nucleation. That is why we choose to represent the fault with a length of 10km (250 Δs), although rupture continues along Homestead Valley well to the north, in a region not of present interest. The end of Kickapoo is offset in a direction perpendicular to Kickapoo by 200m from the Homestead Valley. Thus for the simulation, the center of the Homestead Valley fault is chosen to be at 160 m below the main fault ($y = -4\Delta s$) and at 280 m ahead of its end (x=107Δ). The right half is straight and parallel to the main fault ($125\Delta s$). Along the left one, the orientation of the fault varies from 0^o to 30^o along 2km ($50\Delta s$) to reach the value of 30^o and finally keeps it along 3km($75\Delta s$).
The modelling is represented in the figure 5.1 in the xy plane.
Actually, when a length cannot correspond to reality because of the high dimension of the system (especially for the Johnson Valley and Homestead Valley faults), it is chosen roughly so that when the rupture reaches the end of these faults it does not have a lot of influence on the other tip of the ruptured zone.

5.3 Study of the rupture along the first fault

Figure 5.2: Along Johnson Valley and Kickapoo faults, slip δ (as $D* = 3\mu\delta/(-\sigma_{yy}^0 R_0^o)$ vs. $s* = 3s/R_0^o$ where s is the curvilinear coordinate) for each 0.18s (that is $9R_0^o/c_p$)

Figure 5.3: Along Johnson Valley and Kickapoo faults. Slip velocity v (as $V* = \mu v/(-\sigma_{yy}^0 c_p)$ vs. $s* = 3s/R_0^o$ where s is the curvilinear coordinate) for several time steps $N = 6c_p t/R_0^o$.

The nucleation is simulated at the center of the segment of the Johnson valley fault, that is around the cell -150 (represented by a circle in the figure 5.1. According to the orientation of the fault (26^o) around this location, the equation 2.36 determines the minimum nucleation size as $L_c = 5\Delta s$ that it about $2R_0$, which does not respect the assumption needed to define this quantity (R_0 has to be smaller than the other dimensions). To enable the initiation of the rupture, the length of the initial crack is taken as $L_{coul} = 20\Delta s$. The origin of time is taken once the rupture begins to propagate.

Slip velocity and slip

The rupture propagates bilaterally along first Johnson Valley, continues along the curved part and along the Kickapoo fault. That can be noticed in the figures 5.2 and 5.3 , which represent respectively the slip δ (as $D* = 3\mu\delta/(-\sigma^0_{yy}R^o_0)$ for each 0.18s (that is $9R^o_0/c_p$) and the slip velocity v (as $V* = \mu v/(-\sigma^0_{yy}c_p)$) for several time steps ($N = 6c_p t/R^o_0$), all along the fault (as $s* = 3s/R^o_0$ where s is the curvilinear coordinate). The variation with time of the slip velocity is available at:

http:\\esag.harvard.edu\fliss\JV+Kp_slipvel.avi

Actually the rupture reaches the "left" end of the Johnson Valley fault at N=531 (1.77s). On the other side, it reaches the curved part (at the cell -50) at the time step N=437 (1.47s), the straight part of Kickapoo fault at N=629 (2.1s) and its end at N=1011 (3.37s).

The slip velocity increases slighly along the curved part and it is higher along Kickapoo than along Johnson Valley fault, this is partly because of the decrease of the normal stress along the fault and because the ruptured zone is getting longer. We notice too that, as we wanted at the beginning, reaching the southeast end of Johnson Valley seems to have no influence on the propagation of the rupture on the other side. Besides, after slipping, the end of the Kickapoo fault seems to lock very rapidly and stop slipping, which is represented between the time N=1060 and N=1200 but even after.

According to the representation of slip, figure 5.2, the maximum of slip is approximately 4.4m. The average along Johnson Valley is approximately 3.3m whereas in [9] it is specified as 2.0 ± 0.5m. It is certainly because of the assumption of the slip weakening model. The average along Kickapoo is about 3.6m.

Rupture velocity

Figure 5.4: Position of the left and right ends of the ruptured zone along Johnson Valley and Kickapoo (as $s* = 3s/R^o_0$ where s is the curvilinear coordinate, the origin is the beginning of the part of the fault parallel to the x-axis) for each time step (as $N = 6c_p t/R^o_0$). The two lines at each edge indicate the length of the ruptured zone.The length of the right part of the fault is about 4 km ($s* = 100$) the length of the left part is 11km ($s* = -275$).

As shown in the figures 5.2 and 5.3, the rupture does not seem disturbed by the

Figure 5.5: Rupture velocity v_r (as v_r/c_s) between the initiation of the rupture and the moment when it reaches the end of Kickapoo fault (in terms of time step $N = 6c_pt/R_0^o$) along first the Johnson Valley fault and afterwards Kickapoo fault.

curvature of the fault. This is consistent with the results of Kame in [14] which suggest that for this orientation of the compressive principal stress, the rupture along the branch (Kickapoo fault) is favored.
The figure 5.4 represents the propagation of each tip of the ruptured zone.The velocity of the right tip does not change when it reaches the curved part ($s* = -50$) or the straight part parallel to the x axis ($s* = 0$).
Besides, the rupture velocity v_r, represented in the figure 5.5 (as v_r/c_s) increases and keeps roughly a value along the curved and the straight parts which is around $0.9c_s$.

5.4 Does the rupture jump from the Kickapoo fault to the Homestead Valley fault?

Using the slip history rate of the main fault, we want to know now if a rupture, or more, can nucleate along Homestead Valley fault and if it does, if it propagates bilaterally or not and finally which is the influence of the geometry on the propagation.

Stress distribution around the Homestead Valley fault

Here, we wanted to study the influence of the rupture along the main fault on the stress field around the second fault studied.

□

Figure 5.6: Stress distribution around Homestead Valley fault for elements parallel to it. The contoured quantity $\sigma_{12}/(-\mu_s\sigma_{22})$ (where (x_1, x_2) is the tangential and the normal vector of each element ,figure 1.2) is shown for each element (as $x* = 3x/R_0^o$ and $y* = 3y/R_0^o$). Representation for several time steps: $N = 2c_pt/R_0$

For that we have considered a region with the same orientation as the fault, with the same length and a thickness of 320 m (8cells). The Homestead Valley fault is in the center of this region. The quantity $\sigma_{12}/(-\mu_s\sigma_{22})$ is contoured for several time steps in the figure 5.6 and all along the time of calculations in the movie available at:

http:\\esag.harvard.edu\fliss\HV_stress.avi

First, we notice something which does not happen in the stress distribution around the straight fault for parallel elements descibed in the last chapter: this is the blue region which moves from the time step N=800 to N=1200 and which represents a negative ratio $\sigma_{12}/(-\mu_s\sigma_{22})$. The static study in the first part shows that a tensile region (σ_{22}>0) does not exist. Indeed, the calculations shows that for these regions the shear stress is negative which implies a left lateral slip if the ratio ever becomes more negative than -1 (it is not clear from the figure if it does but other calculations can show that it does in some locations). Nevertheless only right lateral slip is allowed in our calculation.
The critical value of 1 for this ratio (which means that a rupture is possible) is first reached in the curved part around the time step $N = 1040$. The region where the rupture is possible expands especially in the straight part and keeps a constant shape after the time step $N = 1300$ which is shown in the last picture.
Thus, this representation shows that a rupture is likely to nucleate along Homestead Valley fault and perhaps in several location.

Jump of the rupture and bilateral propagation

Figure 5.7: Slip velocity v (as $V* = \mu v/(-\sigma^0_{yy}c_p)$ vs. $x* = 3x/R^o_0$ and $y* = 3y/R^o_0$) for several time steps $N = 4c_pt/R^o_0$,along the faults, around the stepover

Figure 5.8: Slip velocity v (as $V* = \mu v/(-\sigma^0_{yy}c_p)$ vs. $x* = 3x/R^o_0$ and $y* = 3y/R^o_0$) for several time steps $N = 4c_pt/R^o_0$,along the faults

The last calculations lead to the possibility of multiple nucleation along the second fault. However, as a matter of fact, a detailed calculation of the rupture shows that it nucleates at a single location: along the curved part the cell -8 ($x* = 99$ and $y* = -4.18$), which is just below the main fault (in terms of $y*$, the end of Kickapoo fault being $x* = 100$ and $y* = 0$) and which has an orientation of 2.8^o. The nucleation occurs at $N = 1022$ (3.4s) and propagates bilaterally, almost instantaneously at $N = 1028$ (3.43s). The figure 5.7 represents the slip velocity v (as $V* = \mu v/(-\sigma^0_{yy}c_p)$) along Kickapoo and Homestead Valley faults represented in the xy plane around the stepover before and after the jump. The figure 5.8 represents the slip velocity and so the rupture propagation at large scale, along Johnson Valley, then Kickapoo and finally Homestead Valley. The varaitions with time are available respectively at:

http:\\esag.harvard.edu\fliss\JV+Kp+HV_slipvelzoom.avi
http:\\esag.harvard.edu\fliss\JV+Kp+HV_slipvel.avi

Figure 5.9: Position of the left and right ends of the ruptured zone (as $s* = 3s/R_0^o$ where s is the curvilinear coordinate, the origin is the center of Homestead Valley fault) for each time step (as $N = 6c_p t/R_0^o$). The two lines at each edge indicate the length of the ruptured zone. The half length of the fault is fixed to about 5km ($s* = 125$).

The rupture propagates bilaterally : forward along the straight part of Homestead Valley parallel to the Kickapoo fault and backward along the curved part then the straight fault which has a constant orientation of 30^o. Actually the rupture velocity slows down along the curved part as the figure 5.9 shows. The right end of the ruptured zone moves forward more quickly than the left end which has to contend with the curvature. The normal stress increases in this part. Forward, the rupture reaches the end at $N = 1513$ (5.04s). Backward, it finishes crossing the curved part at $N = 1287$ (4.29s). Its velocity increases again (as the discontinuity of the line at the cell -50 suggests) and keeps a constant value along the oblique straight part. It reaches the end at $N = 1648$ (5.49s). Indeed, in the static study, we have shown that the pre stress allows a propagation along the fault with such orientation (section 1.3.3). But it does not correspond to the seismologic observations which suggests that the rupture propagates backward only 4km, [25].

Figure 5.10: Along the Homestead Valley fault, slip velocity v (as $V* = \mu v/(-\sigma_{yy}^0 c_p)$ vs. $s* = 3s/R_0^o$ where s is the curvilinear coordinate) for several time steps $N = 6c_p t/R_0^o$

Thanks to the figure 5.10, we can compare the slip velocity along the different part of the fault. It is higher along the straight part than along the curved part where it decreases dramatically. However when the rupture reaches the oblique straight part, the slip velocity increases again, but keeps lower than the one for an orientation parallel to the x-axis. Besides, as expected, the end of the rupture forward has no influence for the the rupture backward.
After slipping, the ends of the faults seem to lock, as in the self healing pulse. The variation of the slip velocity along this second fault is available at:

http:\\esag.harvard.edu\fliss\HV_slipvel.avi

Figure 5.11: Along the Homestead Valley fault, slip δ (as $D* = 3\mu\delta/(-\sigma_{yy}^0 R_0^o)$ vs. $s* = 3s/R_0^o$ where s is the curvilinear coordinate) for each 0.18s (that is $9R_0^o/c_p$)

Finally, the figure 5.11 represents the slip δ (as $D* = 3\mu\delta/(-\sigma_{yy}^0 R_0^o)$) all along the fault (in terms of cells $s* = 3s/R_0^o$). It is not the location of nucleation which observes the maximum slip (approximately 4m). This one (around the cell -10) corresponds to the beginning of the straight which is close to the nucleation site and on which high shear stress is applied, as the stress distribution around the fault 5.6 suggests. The average of slip is approximately 2.4m. We observe thus the high drop of slip along the curved part. The rupture would stop if the fault does not stop curving. The feature of the slip and slip velocity can be compare to those for the long faults in the last chapter. Thus, this feature is certainly due to the characteristics of initial stress along the curved part. But like explained in the last chapter, the first fault may not favor the propagation of the rupture along it.

Conclusion

In order to unravel the directivity of past large earthquakes rupturing through complex fault systems ([6]), including dynamics jumps and branching, we try to understand the mechanism of "backward branching" (like in the 1992 Landers earthquake). We find that we can numerically reproduce backwards branching only when the rupture stops on the main fault and jumps to the second, non parallel fault, where it nucleates and evolves bilaterally (one part of it being the backward branch). This new general mechanism of a jump followed by a bilateral propagation is responsible for creation of fault segments rupturing backwards by comparison with general dynamic development of the earthquake.

Figure 5.12: Schematic Map of the 28 June 1992, Landers earthquake area. Solid lines is the 28 June, 1992 surface rupture

"Could we infer the directivity of a past earthquake from the geometry of the fault system" (Nakata et al., [17]). This study shows that there is no simple correlation between fault geometry and rupture directivity. Inferring rupture directivity from branch geometry will be possible only when detailed fault characterization in the vicinity of the branch can ascertain whether direct turning of the rupture path through an angle or jumping and propagating bilaterally were more likely involved in prior events. For example, if we use only the geometry of the faults involved in the 1992 Landers earthquake (Figure 5.12) and if we suppose we do not kiow the location of the epicenter (actually on Johnson Valley fault), it is not obvious at all to infer the rupture directivity because of the complexity of this geometry.

Acknowledgements

These studies were supported at Harvard by NSF-EAR award 0105344 on "Rupture propagation and arrest in geometrically complex fault systems", and by a Southern California Earthquake Center (SCEC) grant on "Dynamics of fault branching"; SCEC is supported through USGS cooperative agreement 02HQAG0008 and NSF cooperative agreement EAR-0106924. My gratitude goes out to Professor James R. Rice and Doctor Renata Dmowska for welcoming me at Harvard University for this internship and giving to me the opportunity to work with them on this project. I am glad to have worked with Harsha Bhat. I wish to thank Philippe Moireau for his support and advices. I thank Yajing Liu, Denzil Vaughn, Lorenzo Valdevit and Ken Nahshon for the nice moment in our office and for their technical support. Finally, I want to thank all the people I have met at the university who make my stay very pleasant.

Appendix A

Stress field in a singular crack model

All these results are from Rice [20], or equivalent basic sources on crack theory, but following the particular mode of presentation in Poliakov and al. [19] where some characteristics of analytic functions are used to express linear elastic crack tip stress fields for some elastic crack problems.
By superposition, this configuration of crack and loading may be reduced to the case of forces prescribed on the crack surfaces. Indeed, one first solves the problem without crack and determines the stresses $\sigma_{ij}(x,0)$ on the prospective crack lines,L. Here this is simply the uniform pre-stress. Then the crack problem is solved considering the reverse of these stresses acting on L and other conditions required.

$$\sigma_{ij} = \sigma_{ij}^0 + \Delta\sigma_{ij} \tag{A.1}$$

For an in-plane shear rupture (mode II), just the shear stress applied along the crack has an influence on the stress created by the crack.

$$\begin{aligned}\Delta\sigma_{ij} = \frac{K_{II}}{\sqrt{2\pi r}} &\begin{pmatrix} -\sin(\frac{\theta}{2})(2+\cos(\frac{\theta}{2})\cos(\frac{3\theta}{2})) & \cos(\frac{\theta}{2})(1-\sin(\frac{\theta}{2})\sin(\frac{3\theta}{2})) \\ \cos(\frac{\theta}{2})(1-\sin(\frac{\theta}{2})\sin(\frac{3\theta}{2})) & \sin(\frac{\theta}{2})\cos(\frac{\theta}{2})\cos(\frac{3\theta}{2}) \end{pmatrix} \\ &+ \begin{pmatrix} 0 & -\Delta\tau \\ -\Delta\tau & 0 \end{pmatrix} + O(\sqrt{r}) \\ &\text{where } K_{II} = (\sigma_{xy}^0 - \tau_r)\sqrt{\frac{\pi X}{2}} \text{ is the stress intensity factor} \\ &\text{and} \Delta\tau = \sigma_{xy}^0 - \tau_r \text{ is the stress drop.}\end{aligned} \tag{A.2}$$

Appendix B

Green's functions

The elastodynamic 2-D Green's functions are given by [27]:

$$\begin{aligned}
&G_{11}(\mathbf{x}-\mathbf{x}', t-t') - G_{22}(\mathbf{x}-\mathbf{x}', t-t') \\
&\quad = -\frac{1}{2\pi\mu}(\gamma_2^2-\gamma_1^2)\frac{c_s^2}{r^2}\left[2(t-t')^2 - \frac{r^2}{c_p^2}\right]\frac{1}{\sqrt{(t-t')^2-(r/c_p)^2}}H(t-t'-\frac{r}{c_p}) \\
&\qquad + \frac{1}{2\pi\mu}(\gamma_2^2-\gamma_1^2)\frac{c_s^2}{r^2}\left[2(t-t')^2 - \frac{r^2}{c_s^2}\right]\frac{1}{\sqrt{(t-t')^2-(r/c_s)^2}}H(t-t'-\frac{r}{c_s}), \quad \text{(B.1)}
\end{aligned}$$

$$\begin{aligned}
G_{11}(\mathbf{x}-\mathbf{x}', t-t') + G_{22}(\mathbf{x}-\mathbf{x}', t-t') &= \frac{1}{2\pi\mu}\frac{c_s^2}{c_p^2}\frac{1}{\sqrt{(t-t')^2-(r/c_p)^2}}H(t-t'-\frac{r}{c_p}) \\
&+ \frac{1}{2\pi\mu}\frac{1}{\sqrt{(t-t')^2-(r/c_s)^2}}H(t-t'-\frac{r}{c_s}), \quad \text{(B.2)}
\end{aligned}$$

$$\begin{aligned}
G_{12}(\mathbf{x}-\mathbf{x}', t-t') &= \frac{1}{2\pi\mu}\gamma_1\gamma_2\frac{c_s^2}{r^2}\left[2(t-t')^2 - \frac{r^2}{c_p^2}\right]\frac{1}{\sqrt{(t-t')^2-(r/c_p)^2}}H(t-t'-\frac{r}{c_p}) \\
&- \frac{1}{2\pi\mu}\gamma_1\gamma_2\frac{c_s^2}{r^2}\left[2(t-t')^2 - \frac{r^2}{c_s^2}\right]\frac{1}{\sqrt{(t-t')^2-(r/c_s)^2}}H(t-t'-\frac{r}{c_s}) \quad \text{(B.3)}
\end{aligned}$$

where $r = \|\mathbf{x}-\mathbf{x}'\|$, $\gamma_i = (x_i - x_i')/r$ and $H(\cdot)$ is the Heaviside step function. c_p and c_s denote the P- and S-wave velocities, respectively.
Note that the expression

$$\frac{c_s^2}{r^2}\left(\frac{1}{\sqrt{(t-t')^2-(r/c_p)^2}} - \frac{1}{\sqrt{(t-t')^2-(r/c_s)^2}}\right) \quad \text{(B.4)}$$

converges to a finite limit value as $r \to 0$, so that the elastodynamic Green's functions that contain this expression are not hypersingular at $r = 0$.
The elastostatic 2-D Green's functions are given by :

$$G_{11}(\mathbf{x}-\mathbf{x}') - G_{22}(\mathbf{x}-\mathbf{x}') = \frac{1}{4\pi\mu}\left[-\left(1-\frac{c_s^2}{c_p^2}\right)(\gamma_2^2-\gamma_1^2)\right] \quad \text{(B.5)}$$

$$G_{11}(\mathbf{x}-\mathbf{x}') + G_{22}(\mathbf{x}-\mathbf{x}') = \frac{1}{4\pi\mu}\left[\left(1-\frac{c_s^2}{c_p^2}\right) - 2\left(1+\frac{c_s^2}{c_p^2}\right)\log r\right] \tag{B.6}$$

$$G_{12}(\mathbf{x}-\mathbf{x}') = \frac{1}{4\pi\mu}\left(1-\frac{c_s^2}{c_p^2}\right)\gamma_1\gamma_2 \tag{B.7}$$

as explained in [30].

Appendix C

Finite part of the divergent integral

The divergent integrals appearing in the BIEM have the form:

$$\int_a^b \frac{f(t)}{(b^2 - t^2)^{3/2}} dt \quad (a < b), \tag{C.1}$$

where $t = b$ is the hypersingular point at the wave front and f is a continuous function. This is to be evaluated by taking a finite part. The principle is to isolate the divergent part of the integral using a small ε and taking the limit $\varepsilon \to 0$:

$$\begin{aligned} \int_a^{b-\varepsilon} \frac{f(t)}{(b^2 - t^2)^{3/2}} dt &= -\int_a^{b-\varepsilon} \frac{f(b) - f(t)}{(b^2 - t^2)^{3/2}} dt + f(b) \int_a^{b-\varepsilon} \frac{dt}{(b^2 - t^2)^{3/2}} \\ &= -\int_a^{b-\varepsilon} \frac{f(b) - f(t)}{(b^2 - t^2)^{3/2}} dt - \frac{a}{b^2} \frac{f(b)}{\sqrt{b^2 - a^2}} \\ &\qquad + \frac{1}{\sqrt{\varepsilon}} \frac{(b - \varepsilon) f(b)}{b^2 \sqrt{2b - \varepsilon}} \end{aligned} \tag{C.2}$$

then,

$$\text{p.f.} \quad \int_a^b \frac{f(t)}{(b^2 - t^2)^{3/2}} dt = -\int_a^{b-\varepsilon} \frac{f(b) - f(t)}{(b^2 - t^2)^{3/2}} dt - \frac{a}{b^2} \frac{f(b)}{\sqrt{b^2 - a^2}} \tag{C.3}$$

This is the finite part of the divergent integral first defined by Hadamard(1923). Kame and Yamashita in 13 introduce another definition for the finite part

$$\text{p.f.} \quad \int_0^a x^\alpha \phi(x) dx = \int_{-\varepsilon}^a \phi(x) \left| x^\alpha \right| H(x) dx. \tag{C.4}$$

The integral in the left hand side contains generally a hypersingularity at $x = 0$ for $\alpha < -1$, so that it is divergent in the classical sense. They prove that the result given by the equation C.3 and the one given by this last equation C.4 are equivalent. But it is harder to understand why they use this definition in the calculation of convolution kernels.

Bibliography

[1] Aki, K. and P. Richards, Quantitative seismology (Second Edition), Sausalito, CA: University Science books, 2002.

[2] Andrews, D. J., Rupture velocity of plane strain shear cracks, *J. Geophys. Res, 81*, 5679-5687, 1976.

[3] Cochard, A. and R. Madariaga, Dynamic faulting under rate-dependent friction, *Pure Applied Geophys., 142*, 419-445, 1994.

[4] Das, S. and K. Aki, A numerical study of two-dimensional spontaneous rupture propagation, *Geophys. J. R. Astron. Soc., 50* 643-668, 1977

[5] Das, S. and B. V. Kostrov, On the numerical boundary integral equation method for three-dimensional synamic shear crack models, *Trans. ASME J. Appl. Mech., 54*, 99-104, 1987

[6] Dmowska,R., J. R. Rice and N. Kame, Fault branching and rupture directivity, *EOS Trans. AGU*, Fall Meet. Suppl., Abstract NG21B-0939,2002.

[7] Felzer, K. R. and G. C. Beroza, Deep structure of a fault discontinuity, *Geophysical Research Letters, 26*, 2121-2124, 1999.

[8] Fossum, A. F., and L. B. Freund, Nonuniformly moving shear crack model of a shallow focus earthquake mechanism, *J. Geophys. Res, 80*, 3343-3347, 1975.

[9] Hardebeck, J. L. and E. Hauksson, Crustal stress field in Southern California and its implications for fault mechanics, *J. Geophys. Res., 106*, 21,859-21, 882, 2001.

[10] Harris, R. A., and S. M. Day, Dynamics of Fault Interaction: Parallel Strike-Slip Faults, *J. Geophys. Res., 98*, 4461-4472, 1993.

[11] Ida, Y.,Cohesive force accross the tip of a longitudinal-shear crack and Griffith's specific surface energy, *J. Geophys. Res., 77*, 3796-3805, 1972

[12] Johnson, E., On the initiation of unidirectional slip, *Geophy. J. Int., 101*, 125-132, 1990

[13] Kame, N. and T. Yamashita, Simulation of the spontaneous growth of a dynamic crack without constraints on the crack tip path, *Geophys. J. Int, 139*, 345-358, 1999

[14] Kame, N., J.R. Rice and R. Dmowska, Effects of pre-stress state and rupture velocity on dynamic fault branching,*J. Geophys. Res., 108(B5)*, cn:2265, doi:10.1029/2002JB002189, pp. ESE 13-1 to 13-21, 2003. Pre-publication version: http:\\esag.harvard.edu\dmowska\KRD.pdf

[15] Koller, M., M. Bonnet and R. Madariaga, Modelling of dynamical crack propagation using regularized time-domain boundary integral equation, *Wave Motion, 16*, 339-366, 1992

[16] Li, Y.-G, K. Aki, D. Adams and A. Hasemi, Seismic guided waves trapped in the fault zone of the Landers, California, earthquake of 1992, *J. Geophys. Res.,99*, 11,705-11, 722, 1994.

[17] Nakata, T., K. Shimazaki, Y. Suzuki, and E. Tsukuda, Fault branching and directivity of rupture propagation, (in Japanese), *J. Geography, 107*, 512-528, 1998.

[18] Palmer, A. C. and J. R. Rice, The growth of slip surfaces in the progressive failure of over-consolidated clay, *Proc. Roy. Soc. London., A332* 527-548, 1973

[19] Poliakov, A. N. B., R. Dmowska and J.R. Rice, Dynamic shear rupture interactions with fault bends and off-axis secondary faulting, *J. Geophys. Res., 107(B11),cn:*2295,doi:10.1029/2001JB000572, pp. ESE 6-1 to 6-18, 2002. Pre-publication version:
http:\\esag.harvard.edu\dmowska\PDR.pdf

[20] Rice, J. R., Mathematical analysis in the mechanics of fracture, in Fracture, an Advanced Treatise, VolumeII,*ed. H. Liebowitz*, Academic Press, New York, 1968

[21] Rice. J. R., The mechanics of earthquake rupture, *Physics of Earth's interior* (Proc. Int'l. School of Physics 'Enrico Fermi', Course 78, pp. 555-649, 1979; ed. A. M. Dzieswonski and E. Boschi), Italian Physical Society and North Holland Publ. Co., Amsterdam, 1980.

[22] Rice, J. R., Notes on elastodynamics, Green's functions, response to transformation strain and crack or fault sources, for use in teaching on earthquake, source processes and adavnced elasticity, 1998. Available at: http:\\esag.harvard.edu\rice\e1_Elastodyn_Source_Theory.pdf

[23] Rice, J. R., C. G. Sammis and R. Parsons, Off-fault secondary induced by a dynamic slip pulse, *submitted to Bull. Seismol. Soc. Amer*, August 2003

[24] Seelig, Th. and D. Gross, Analysis of dynamic crack propagation using a time domain boundary integral equation method, *Int. J. Solids Structures, vol. 34, No. 17*, 2087-2103, 1997.

[25] Sowers, J. M., J. R. Unruh, W. R. Lettis, and T. D. Rubin, Relationship of the Kickapoo fault to the Jonhson Valley and Homestead Valley Faults, San Bernadino County, California, *Bull. Seismol. Soc. Am., 84*, 528-536, 1994

[26] Spotila, J. A. and K. E. Sieh, Geological investigations of a "slip gap" in the surficial ruptures of the 1992 Landers earthquake, southern California, *J. Geophys. Res., 100*, 543-559, 1995

[27] Tada, T. and T. Yamashita, Non hypersingular boundary integral equations for two dimensional non-planar crack analysis, *Geophys. J. Int., 130*, 269-282, 1997.

[28] Yamashita, T. and E. Fukuyama, Apparent critical slip displacement caused by the existence of a fault zone, *Geophy. J. Int., 125*, 459-472, 1996

[29] Zachariasen, J. and K. Sieh, The transfer of slip between two en echelon strike-slip faults: a case study from the 1992 Landers earthquake, southern California, *J. Geophys. Res., 100*, 15, 281-15, 302, 1995.

[30] Zhang, Ch. and J. D. Achenbach, A new boundary integral equation formulation for elastodynamic and elastostatic crack analysis, *Trans. ASME J. Appl. Mech, 56*, 284-290, 1989