Doctoral research project on

Theoretical and computational aspects of a fast method for elastic waveform tomography

Advisors: Marc Bonnet (POEMS), Bojan Guzina (CEGE).

Academic units: POEMS (ENSTA, Université Paris Saclay), CEGE (University of Minnesota).

The concept of topological derivative (TD) provides an effective approach to elastic waveform tomography with important applications including non destructive material testing, seismic and medical imaging. It falls within the category of qualitative approaches to inverse scattering [3, 5], which also include e.g. the linear sampling method.

TD-based imaging techniques are fast (thanks to their non-iterative nature) and quite flexible in terms of testing configurations. They are also easy to implement using standard (e.g. FEM-based) computational platforms once the formulation of the imaging functional is established (this is normally done using asymptotic analysis of the field scattered by a vanishing trial defect).

TD-based imaging may be developed and applied under conditions corresponding to either moderate or high frequencies. Both the behavior of TD functionals and their mathematical justification are quite different in these cases. The proposed research aims at obtaining insight into the properties, capabilities and limitations of elastodynamic TD imaging functionals for both frequency regimes, thereby gaining perspective on their performance through a combination of diverse mathematical and computational tools.

TD imaging: moderate frequencies. When applied to the imaging of defects whose diameter is comparable to the characteristic wavelength, TD-based maps can be interpreted in a heuristic way: spatial locations where the TD is most negative are such that the nucleation of a small trial defect triggers the sharpest decrease of a misfit cost functional associated with the imaging problem, and so canvas the image of a defect. Such interpretation is supported by mounting computational and experimental evidence. The above-outlined heuristic assumes the TD function to have (i) largest magnitude and (ii) negative sign at the location of an actual defect. While intuitively natural, such behavior largely remains to be justified, especially in the case of elastic (e.g. ultrasonic or seismic) waves. Accordingly, the aim of this project is to fill the gap by investigating the mathematical properties of elastodynamic TD-based imaging, starting with idealized conditions that are both geometrically simple and data-rich, and then considering increasingly ”harsher” testing configurations. This work will rely on, and extend, related previous work, e.g. [2] (and ongoing work with F. Cakoni, Rutgers U.) for linear acoustics using far-field data or [1] for the identification of small objects.

TD imaging: high frequencies. There is a mounting evidence, drawn from both laboratory and numerical experiments, that an elastodynamic statement of the TD indicator exhibits strong localization along the boundary of discrete material heterogeneities (e.g. defects) at higher interrogation frequencies. In particular it is observed that for wavelengths that are smaller than the characteristic obstacle size, extreme negative values of the TD functional are confined to a narrow region (fraction of the wavelength in thickness) tracking the boundary of an obstacle to be identified from the data. Recently, such behavior has been supported theoretically in the context of acoustic waves by way of (i) multipole expansion, (ii) Kirchhoff scattering approximation, and (iii) catastrophe theory [4]. However, an asymptotic analysis of the high-frequency elastodynamic TS behavior is still lacking. One key challenge in such setting is the structure of the governing Navier equation which allows for both compressional, shear, and surface waves. As a result the goal is to establish a high-frequency TD analysis, including an asymptotic study of the germane diffraction integrals, that accounts for an intrinsic interplay between multiple wave types a phenomenon absent in optics that most high-frequency analyses cater for.

Goals. In this project, particular items of interest (in both regimes) include the range of configurations amenable to theoretical justification, and the spatial resolution of the TD imaging functional. The theoretical findings will be demonstrated on numerical experiments, designed in such a way to both confirm the behavior of TD-based imaging functionals predicted by mathematical analyses and experimentally determine the extent to which theoretical limitations can be transcended in practice. The University of Minnesota is also home to a world-class Waves and Imaging laboratory (see `http://www.bojanguzina.org/`) where many of the theoretical developments can be verified [6].

Information for prospective applicants

This project is to be undertaken within the framework of the joint Université Paris Saclay - University of Minnesota PhD program. This program (*cotutelle*) is conducted in English. Each PhD student has two doctoral advisers (one from each institution), and approximately splits his or her time between Paris and Minneapolis metropolitan areas. The duration of a PhD thesis under this program is usually limited to three years, which assumes that candidates have adequate academic preparation (typically in the form of an MS degree in a closely related area). The preferred academic background of applicants should include elasticity, wave propagation and scattering, integral equation methods, and a solid knowledge of applied mathematics. For successful PhD applicant(s), the proposed project offers a dual benefit of immediate technological relevance (e.g. non-destructive material testing) and potential for significant new theoretical developments at the interface of mathematics and solid mechanics. The interdisciplinary nature of this project is reflected in the complementary expertise of the host departments.

For inquiries, please send us a detailed curriculum vitae, including relevant transcripts and a suggested list of names (with email addresses) of reference letter writers.

- Bojan Guzina, Shimizu Professor, Dept. of Civil, Environmental and Geo- Engineering, University of Minnesota, USA. `guzin001@umn.edu`, `http://www.bojanguzina.org/`
- Marc Bonnet, CNRS senior scientist, POEMS, UMA (Dept. of Appl. Math.), ENSTA, France. `mbonnet@ensta.fr`, `http://uma.ensta.fr/~mbonnet`

University of Minnesota (USA): `https://twin-cities.umn.edu/`
Université Paris Saclay (France): `https://www.universite-paris-saclay.fr/en/`

References

[1] Ammari, H and. Garnier, J., Jugnon, V., Kang, H. Stability and resolution analysis for a topological derivative based imaging functional. *SIAM J. Contr. Opt.*, **50**, 48–76 (2012).

[2] Bellis, C., Bonnet, M., Cakoni, F. Acoustic inverse scattering using topological derivative of far-field measurements-based L^2 cost functionals. *Inverse Problems*, **29**, 075012 (2013).

[3] Cakoni, F., Colton, D. *A qualitative approach to inverse scattering theory*. Springer-Verlag (2014).

[4] Guzina, B. B, Pourahmadian, F. Why the high-frequency inverse scattering by topological sensitivity may work. vol. 471, p. 20150187 (2015).

[5] Potthast, R. A survey on sampling and probe methods for inverse problems. *Inverse Problems*, **22**, R1–R47 (2006).

[6] Tokmashev, R., Tixier, A., Guzina, B. Experimental validation of the topological sensitivity approach to elastic-wave imaging. *Inverse Problems*, **29**, 125005 (2013).