Decomposition/Coordination Methods for Multistage Stochastic Optimization Problems

P. Carpentier, J-Ph. Chancelier,
M. De Lara, V. Leclère

École des Ponts ParisTech and ENSTA ParisTech

PGMO Days, 21 novembre 2018

Lecture outline

Outline of the presentation

A long-term effort in our group (I)

1976 A. Benveniste, P. Bernhard, G. Cohen, "On the decomposition of stochastic control problems", *IRIA-Laboria research report* No. 187, 1976

1996 P. Carpentier, G. Cohen, J.-C. Culioli, A. Renaud, "Stochastic optimization of unit commitment: a new decomposition framework" *IEEE Transactions on Power Systems*, Vol. 11, No. 2, 1996

2006 C. Strugarek, "Approches variationnelles et autres contributions en optimisation stochastique", *Thèse de l'ENPC*, mai 2006

2010 K. Barty, P. Carpentier, P. Girardeau, "Decomposition of large-scale stochastic optimal control problems" *RAIRO Operations Research*, Vol. 44, No. 3, 2010

A long-term effort in our group (II)

2013 J.-C. Alais, "Risque et optimisation pour le management d'énergies", *Thèse de l'Université Paris-Est*, décembre 2013

2014 V. Leclère, "Contributions to decomposition methods in stochastic optimization", *Thèse de l'Université Paris-Est*, juin 2014.

2014 M. De Lara, P. Carpentier, J.-P. Chancelier, V. Leclère, "Optimization Methods for the Smart Grid", *report commissioned by Conseil Français de l'Energie*, octobre 2014

2017 P. Carpentier, G. Cohen, "Décomposition-coordination en optimisation déterministe et stochastique", *Springer*, 2017

2018 F. Pacaud, "Optimisation décentralisée pour l'efficacité énergétique", *Thèse de l'Université Paris-Est*, octobre 2018

A long-term effort in our group (III)

2016 M. De Lara, V. Leclère, "Building Up Time-Consistency for Risk Measures and Dynamic Optimization", *European Journal of Operations Research*, Volume 249, Issue 1, pp 177–187, 2016

2017 J.-C. Alais, P. Carpentier, M. De Lara, "Multi-usage hydropower single dam management: chance-constrained optimization and stochastic viability", *Energy Systems* Volume 8, Issue 1, pp 7–30, February 2017

2018 H. Gérard, "Décomposition de problèmes d'optimisation stochastique de grande dimension, avec mesure de risque", *Thèse de l'Université Paris-Est*, octobre 2018

Outline of the presentation

Decomposition-coordination: divide and conquer

- Spatial decomposition
  - Multiple players with their local information
  - Network with decision-makers located at nodes where they control local storage and flows through edges
- Temporal decomposition
  - A state is an information summary
  - Time coordination realized through dynamic programming, by value functions
  - Hard nonanticipativity constraints
- Scenario decomposition
  - Along each scenario, sub-problems are deterministic (powerful algorithms)
  - Scenario coordination realized through Progressive Hedging, by updating nonanticipativity multipliers
  - Soft nonanticipativity constraints

Let us fix problem and notations

$$\min_{\mathbf{U},\mathbf{X}} \overbrace{\mathbb{E}}^{\text{"risk-neutral"}} \left[\sum_{i=1}^{N} \left(\sum_{t=0}^{T-1} L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) + K^i(\mathbf{X}_T^i) \right) \right]$$

subject to dynamics constraints

$$\underbrace{\mathbf{X}_{t+1}^i}_{\text{state}} = f_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \underbrace{\mathbf{W}_{t+1}}_{\text{uncertainty}}) \ , \ \ \mathbf{X}_0^i = f_{-1}^i(\mathbf{W}_0)$$

to measurability constraints on the control $\mathbf{U}_t^i$

$$\sigma(\mathbf{U}_t^i) \subset \sigma(\mathbf{W}_0, \ldots, \mathbf{W}_t) \iff \mathbf{U}_t^i = \mathbb{E}\left(\mathbf{U}_t^i \;\middle|\; \mathbf{W}_0, \ldots, \mathbf{W}_t\right)$$

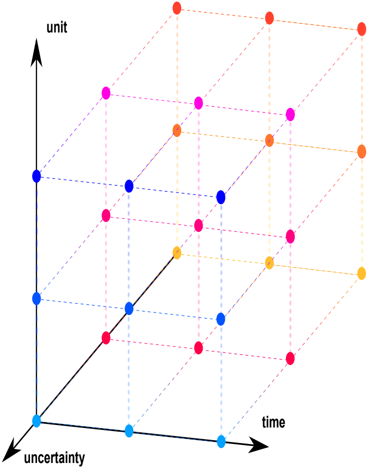
and to instantaneous coupling constraints

$$\sum_{i=1}^{N} Y_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i) = 0$$

(The letter U stands for the Russian word for control: *upravlenie*)

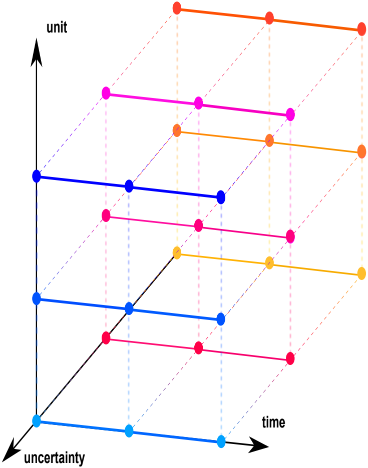
Outline of the presentation

Couplings for stochastic problems



$$\min \sum_{\omega} \sum_{i} \sum_{t} \pi_\omega L_t^i\big(\mathbf{X}_t^i(\omega), \mathbf{U}_t^i(\omega), \mathbf{W}_{t+1}(\omega)\big)$$

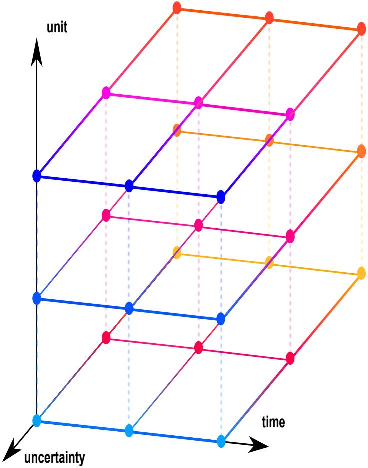
Couplings for stochastic problems: in time



$$\min \sum_{\omega} \sum_{i} \sum_{t} \pi_\omega L_t^i\big(\mathbf{X}_t^i(\omega), \mathbf{U}_t^i(\omega), \mathbf{W}_{t+1}(\omega)\big)$$

s.t. $\mathbf{X}_{t+1}^i = f_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1})$

Couplings for stochastic problems: in uncertainty

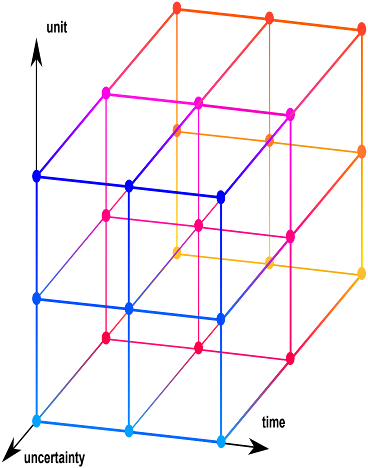


$$\min \sum_{\omega} \sum_{i} \sum_{t} \pi_\omega L_t^i(\mathbf{X}_t^i(\omega), \mathbf{U}_t^i(\omega), \mathbf{W}_{t+1}(\omega))$$

s.t. $\mathbf{X}_{t+1}^i = f_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1})$

$$\mathbf{U}_t^i = \mathbb{E}\left(\mathbf{U}_t^i \;\middle|\; \mathbf{W}_0, \dots, \mathbf{W}_t\right)$$

Couplings for stochastic problems: in space



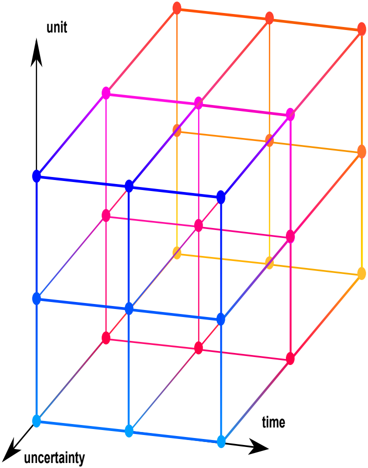
$$\min \sum_{\omega} \sum_{i} \sum_{t} \pi_\omega L_t^i\big(\mathbf{X}_t^i(\omega), \mathbf{U}_t^i(\omega), \mathbf{W}_{t+1}(\omega)\big)$$

s.t. $\mathbf{X}_{t+1}^i = f_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1})$

$$\mathbf{U}_t^i = \mathbb{E}\Big(\mathbf{U}_t^i \;\Big|\; \mathbf{W}_0, \dots, \mathbf{W}_t\Big)$$

$$\sum_i Y_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i) = 0$$

Can we decouple stochastic problems?



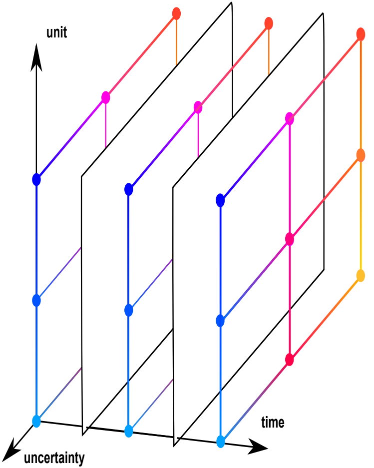
$$\min \sum_{\omega} \sum_{i} \sum_{t} \pi_\omega L_t^i\big(\mathbf{X}_t^i(\omega), \mathbf{U}_t^i(\omega), \mathbf{W}_{t+1}(\omega)\big)$$

$$\text{s.t. } \mathbf{X}_{t+1}^i = f_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1})$$

$$\mathbf{U}_t^i = \mathbb{E}\left(\mathbf{U}_t^i \;\middle|\; \mathbf{W}_0, \ldots, \mathbf{W}_t\right)$$

$$\sum_i Y_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i) = 0$$

Sequential decomposition in time



$$\min \sum_{\omega} \sum_{i} \sum_{t} \pi_\omega L_t^i\big(\mathbf{X}_t^i(\omega), \mathbf{U}_t^i(\omega), \mathbf{W}_{t+1}(\omega)\big)$$

$$\text{s.t.} \quad \mathbf{X}_{t+1}^i = f_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1})$$

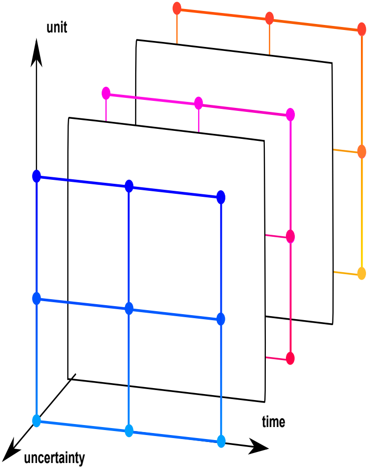
$$\mathbf{U}_t^i = \mathbb{E}\left(\mathbf{U}_t^i \,\middle|\, \mathbf{W}_0, \dots, \mathbf{W}_t\right)$$

$$\sum_i Y_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i) = 0$$

Dynamic Programming

Bellman (56)

Parallel decomposition in uncertainty/scearios



$$\min \sum_{\omega} \sum_{i} \sum_{t} \pi_\omega L_t^i\big(\mathbf{X}_t^i(\omega), \mathbf{U}_t^i(\omega), \mathbf{W}_{t+1}(\omega)\big)$$

$$\text{s.t. } \mathbf{X}_{t+1}^i = f_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1})$$

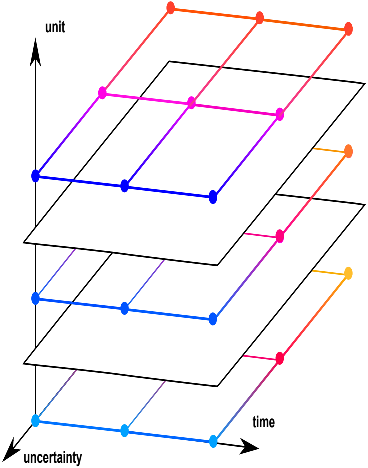
$$\mathbf{U}_t^i = \mathbb{E}\Big(\mathbf{U}_t^i \;\Big|\; \mathbf{W}_0, \dots, \mathbf{W}_t\Big)$$

$$\sum_{i} Y_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i) = 0$$

Progressive Hedging

Rockafellar - Wets (91)

Parallel decomposition in space/units



$$\min \sum_{\omega} \sum_{i} \sum_{t} \pi_\omega L_t^i(\mathbf{X}_t^i(\omega), \mathbf{U}_t^i(\omega), \mathbf{W}_{t+1}(\omega))$$

s.t. $\mathbf{X}_{t+1}^i = f_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1})$

$$\mathbf{U}_t^i = \mathbb{E}\Big(\mathbf{U}_t^i \;\Big|\; \mathbf{W}_0, \ldots, \mathbf{W}_t\Big)$$

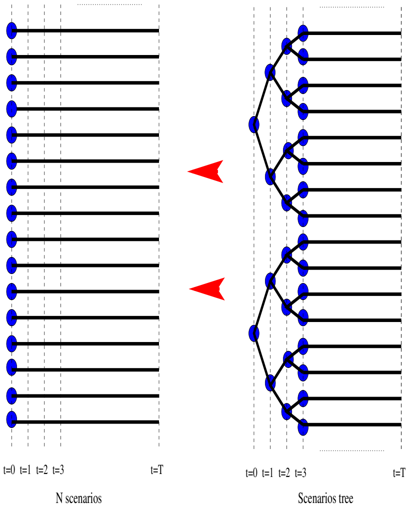
$$\sum_i Y_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i) = 0$$

Price and Quantity
Decompositions with DP

Outline of the presentation

Outline of the presentation

Non-anticipativity constraints are linear



- From tree to scenarios (fan)
- Equivalent formulations of the non-anticipativity constraints
  - pairwise equalities
  - all equal to their mathematical expectation
- Linear structure

$$\mathbf{U}_t = \mathbb{E}\Big(\mathbf{U}_t \;\Big|\; \mathbf{W}_0, \ldots, \mathbf{W}_t\Big)$$

Progressive Hedging stands as a scenario decomposition method

We dualize the non-anticipativity constraints

- When the criterion is strongly convex, we use a Lagrangian relaxation (algorithm "à la Uzawa") to obtain a scenario decomposition
- When the criterion is linear, Rockafellar - Wets (91) propose to use an augmented Lagrangian, and obtain the Progressive Hedging algorithm

Data: step $\rho > 0$, initial multipliers $\left\{\lambda_s^{(0)}\right\}_{s\in\mathbb{S}}$ and mean first decision $\overline{\mathbf{x}}^{(0)}$;

Result: optimal first decision $\mathbf{x}$;

repeat

forall *scenarios* $s \in \mathbb{S}$ **do**

Solve the deterministic minimization problem for scenario s, with a penalization $+\lambda_s^{(k)}\left(\mathbf{x}_s^{(k+1)} - \overline{\mathbf{x}}^{(k)}\right)$, and obtain optimal first decision $\mathbf{x}_s^{(k+1)}$;

Update the mean first decisions

$$\overline{\mathbf{x}}^{(k+1)} = \sum_{s\in\mathbb{S}} \pi_s \mathbf{x}_s^{(k+1)} ;$$

Update the multiplier by

$$\lambda_s^{(k+1)} = \lambda_s^{(k)} + \rho\big(\mathbf{x}_s^{(k+1)} - \overline{\mathbf{x}}^{(k+1)}\big) , \ \forall s \in \mathbb{S} ;$$

until $\mathbf{x}_s^{(k+1)} - \sum_{s'\in\mathbb{S}} \pi_{s'} \mathbf{x}_{s'}^{(k+1)} = 0 , \ \forall s \in \mathbb{S}$;

Outline of the presentation

Suppose you had to manage a day-ahead energy market
You would have to fix reserves by night
and adjust in the morning with recourse energies

From linear to stochastic programming

- The linear program

$$\min_{x \in \mathbb{R}^n} \langle c \,, x \rangle$$
$$Ax + b \quad \geq 0 \quad (\in \mathbb{R}^m)$$

- becomes a stochastic program

$$\min_{x \in \mathbb{R}^n} \sum_{s \in \mathbb{S}} \pi_s \langle c_s \,, x \rangle$$
$$A_s x + b_s \quad \geq 0 \,, \quad \forall s \in \mathbb{S}$$

- We observe that there are as many (vector) inequalities as there are possible scenarios $s \in \mathbb{S}$

$$A_s x + b_s \geq 0 \,, \quad \forall s \in \mathbb{S}$$

and these inequality constraints can delineate an empty domain for optimization

Recourse variables need be introduced for feasability issues

- We denote by $s \in \mathbb{S}$ any possible value of the random variable ξ,
  with corresponding probability π_s
- and we introduce a recourse variable $y = (y_s)_{s \in \mathbb{S}}$ and the program

$$
\begin{aligned}
\min_{x,(y_s)_{s\in\mathbb{S}}} \sum_{s\in\mathbb{S}} \pi_s \Big(\langle c_s \,, x \rangle + \langle p_s \,, y_s \rangle \Big) \\
y_s \quad &\geq 0 \,, \quad \forall s \in \mathbb{S} \\
A_s x + b_s + y_s \quad &\geq 0 \,, \quad \forall s \in \mathbb{S}
\end{aligned}
$$

- so that the inequality $A_s x + b_s + y_s \geq 0$ is now possible, at (unitary recourse) price vector $p = (p_s, s \in \mathbb{S})$
- Observe that such stochastic programs are huge problems, with solution $\big(x, (y_s)_{s\in\mathbb{S}}\big)$, but remain linear

Minimizing the Tail Value at Risk of costs: linear programming formulation

- The risk-averse stochastic linear program with recourse

$$\min_{x,(y_s)_{s\in\mathbb{S}}} \min_{r\in\mathbb{R}} \left\{ r + \frac{1}{1-\lambda} \sum_{s\in\mathbb{S}} \pi_s \Big(\langle c_s \,, x \rangle + \langle p_s \,, y_s \rangle \Big)_+ \right\}$$

- can be written as the linear program

$$\begin{aligned}
\min_{x,(y_s)_{s\in\mathbb{S}}} \min_{r} \min_{(v_s)_{s\in\mathbb{S}}} \quad & r + \frac{1}{1-\lambda} \sum_{s\in\mathbb{S}} \pi_s v_s \\
v_s - \langle c_s \,, x \rangle - \langle p_s \,, y_s \rangle \quad & \geq 0 \,, \quad \forall s \in \mathbb{S} \\
v_s \quad & \geq 0 \,, \quad \forall s \in \mathbb{S} \\
y_s \quad & \geq 0 \,, \quad \forall s \in \mathbb{S} \\
A_s x + b_s + y_s \quad & \geq 0 \,, \quad \forall s \in \mathbb{S}
\end{aligned}$$

Minimizing a mixture: linear programming formulation

- The risk-averse stochastic linear program with recourse

$$\min_{x,(y_s)_{s\in\mathbb{S}}} \min_{r\in\mathbb{R}} \Big\{ \theta \sum_{s\in\mathbb{S}} \pi_s \Big(\langle c_s \, , x \rangle + \langle p_s \, , y_s \rangle \Big) + (1-\theta) r + \frac{1-\theta}{1-\lambda} \sum_{s\in\mathbb{S}} \pi_s \Big(\langle c_s \, , x \rangle + \langle p_s \, , y_s \rangle \Big)_+ \Big\}$$

- can be written as the linear program

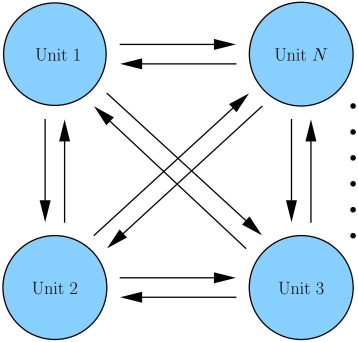
$$\min_{x,(y_s)_{s\in\mathbb{S}}} \min_{r} \min_{(u_s,v_s)_{s\in\mathbb{S}}} \sum_{s\in\mathbb{S}} \pi_s \Big\{ \theta u_s + (1-\theta) r + \frac{1-\theta}{1-\lambda} v_s \Big\}$$

$$\begin{aligned}
u_s - \langle c_s \, , x \rangle - \langle p_s \, , y_s \rangle &\geq 0 \, , \quad \forall s \in \mathbb{S} \\
v_s - u_s + r &\geq 0 \, , \quad \forall s \in \mathbb{S} \\
v_s &\geq 0 \, , \quad \forall s \in \mathbb{S} \\
y_s &\geq 0 \, , \quad \forall s \in \mathbb{S} \\
A_s x + b_s + y_s &\geq 0 \, , \quad \forall s \in \mathbb{S}
\end{aligned}$$

Outline of the presentation

Outline of the presentation

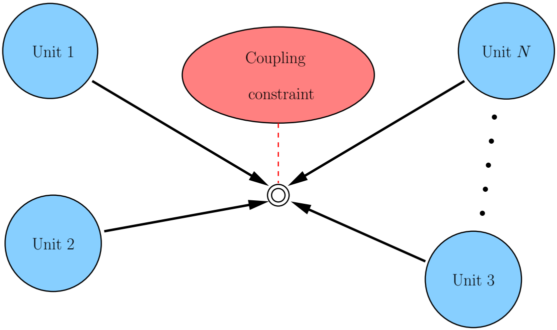
Decomposition and coordination



Interconnected units

- The system to be optimized consists of interconnected subsystems
- We want to use this structure to formulate optimization subproblems of reasonable complexity
- But the presence of interactions requires a level of coordination
- Coordination iteratively provides a local model of the interactions for each subproblem
- We expect to obtain the solution of the overall problem by concatenation of the solutions of the subproblems

Example: the "flower model"

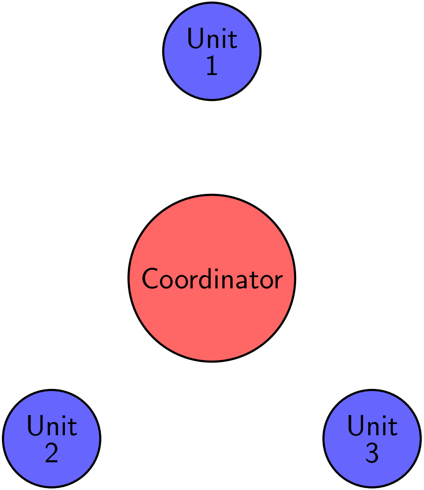


$$\min_u \sum_{i=1}^{N} J_i(u_i)$$

$$\text{s.t.} \quad \sum_{i=1}^{N} \theta_i(u_i) = 0$$

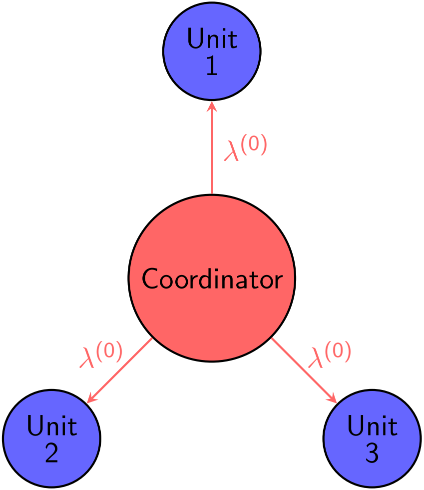
Unit Commitment Problem

Intuition of spatial decomposition



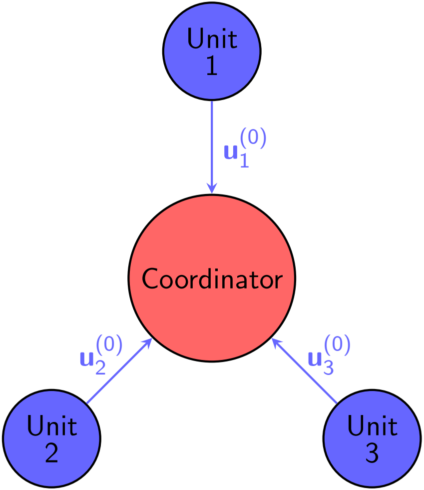
- Purpose: satisfy a demand with N production units, at minimal cost
- Price decomposition

Intuition of spatial decomposition



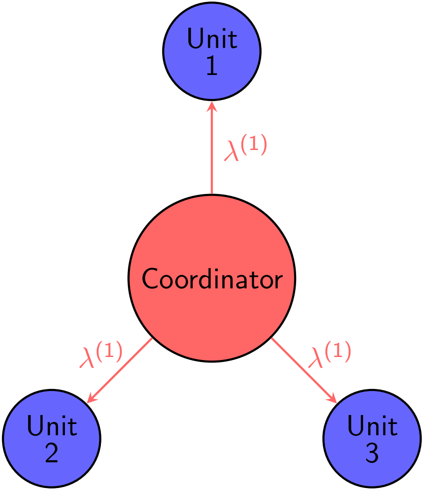
- Purpose: satisfy a demand with N production units, at minimal cost
- Price decomposition
  - the coordinator sets a price λ

Intuition of spatial decomposition



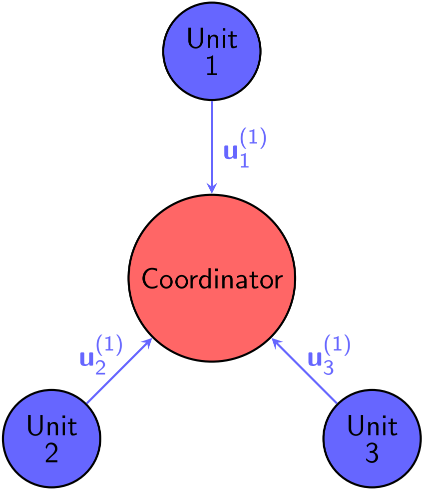
- Purpose: satisfy a demand with N production units, at minimal cost
- Price decomposition
  - the coordinator sets a price λ
  - the units send their optimal decision $\mathbf{u}_i$

Intuition of spatial decomposition



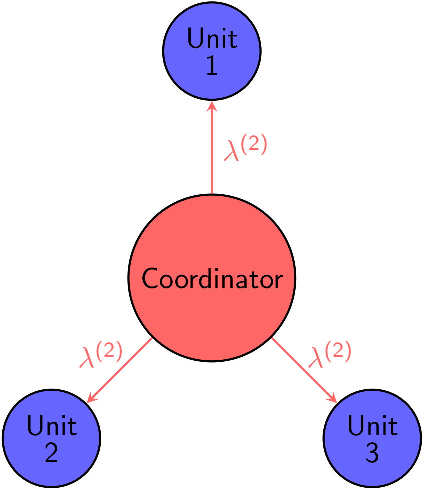
- Purpose: satisfy a demand with N production units, at minimal cost
- Price decomposition
  - the coordinator sets a price λ
  - the units send their optimal decision $\mathbf{u}_i$
  - the coordinator compares total production $\sum_{i=1}^{N} \theta_i(u_i)$ and demand, and then updates the price accordingly

Intuition of spatial decomposition



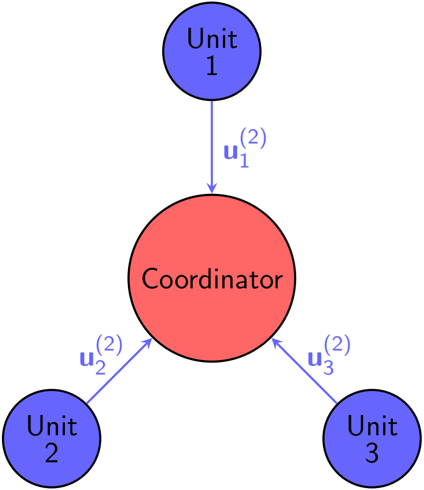
- Purpose: satisfy a demand with N production units, at minimal cost
- Price decomposition
  - the coordinator sets a price λ
  - the units send their optimal decision $\mathbf{u}_i$
  - the coordinator compares total production $\sum_{i=1}^{N} \theta_i(u_i)$ and demand, and then updates the price accordingly
  - and so on...

Intuition of spatial decomposition



- Purpose: satisfy a demand with N production units, at minimal cost
- Price decomposition
  - the coordinator sets a price λ
  - the units send their optimal decision $\mathbf{u}_i$
  - the coordinator compares total production $\sum_{i=1}^{N} \theta_i(u_i)$ and demand, and then updates the price accordingly
  - and so on...

Intuition of spatial decomposition



- Purpose: satisfy a demand with N production units, at minimal cost
- Price decomposition
  - the coordinator sets a price λ
  - the units send their optimal decision $\mathbf{u}_i$
  - the coordinator compares total production $\sum_{i=1}^{N} \theta_i(u_i)$ and demand, and then updates the price accordingly
  - and so on...

Price decomposition relies on dualization

$$\min_{u_i \in \mathcal{U}_i, i=1 \ldots N} \sum_{i=1}^{N} J_i(u_i) \quad \text{subject to} \quad \sum_{i=1}^{N} \theta_i(u_i) = 0$$

1. Form the Lagrangian and assume that a saddle point exists

$$\max_{\lambda \in \mathcal{V}} \min_{u_i \in \mathcal{U}_i, i=1 \ldots N} \sum_{i=1}^{N} \Big(J_i(u_i) + \left\langle \lambda , \theta_i(u_i) \right\rangle \Big)$$

2. Solve this problem by the dual gradient algorithm "à la Uzawa"

$$u_i^{(k+1)} \in \arg\min_{u_i \in \mathcal{U}_i} J_i(u_i) + \left\langle \lambda^{(k)} , \theta_i(u_i) \right\rangle , \quad i = 1 \ldots , N$$

$$\lambda^{(k+1)} = \lambda^{(k)} + \rho \sum_{i=1}^{N} \theta_i(u_i^{(k+1)})$$

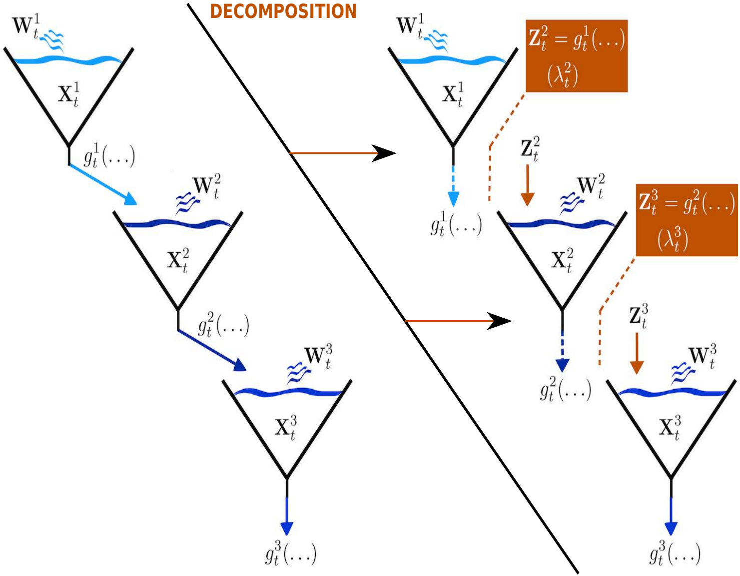
Remarks on decomposition methods

- The theory is available for infinite dimensional Hilbert spaces, and thus applies in the stochastic framework, that is, when the $\mathcal{U}_i$ are spaces of random variables
- The minimization algorithm used for solving the subproblems is not specified in the decomposition process
- New variables $\lambda^{(k)}$ appear in the subproblems arising at iteration k of the optimization process

$$\min_{u_i \in \mathcal{U}_i} J_i(u_i) + \left\langle \lambda^{(k)} , \theta_i(u_i) \right\rangle$$

- These variables are fixed when solving the subproblems, and do not cause any difficulty, at least in the deterministic case

Price decomposition applies to various couplings



Outline of the presentation

Stochastic optimal control (SOC) problem formulation

Consider the following SOC problem

$$\min_{\mathbf{U},\mathbf{X}} \mathbb{E}\Big(\sum_{i=1}^{N} \Big(\sum_{t=0}^{T-1} L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) + K^i(\mathbf{X}_T^i) \Big) \Big)$$

subject to the constraints

$$\mathbf{X}_0^i = f_{-1}^i(\mathbf{W}_0) \ , \qquad i = 1 \ldots N$$

$$\mathbf{X}_{t+1}^i = f_t^i\big(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}\big) \ , \qquad t = 0 \ldots T-1 \ , \ \ i = 1 \ldots N$$

$$\sigma\big(\mathbf{U}_t^i\big) \subset \sigma\big(\mathbf{W}_0, \ldots, \mathbf{W}_t\big) \ , \ \ t = 0 \ldots T-1 \ , \ \ i = 1 \ldots N$$

$$\sum_{i=1}^{N} \theta_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i) = 0 \ , \qquad t = 0 \ldots T-1$$

Stochastic optimal control (SOC) problem formulation

Consider the following SOC problem

$$\min_{\mathbf{U},\mathbf{X}} \sum_{i=1}^{N} \left(\mathbb{E}\Big(\sum_{t=0}^{T-1} L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) + K^i(\mathbf{X}_T^i) \Big) \right)$$

subject to the constraints

$$\mathbf{X}_0^i = f_{-1}^i(\mathbf{W}_0) \ , \qquad i = 1 \dots N$$

$$\mathbf{X}_{t+1}^i = f_t^i\big(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}\big) \ , \qquad t = 0 \dots T-1 \ , \ \ i = 1 \dots N$$

$$\sigma\big(\mathbf{U}_t^i\big) \subset \sigma\big(\mathbf{W}_0, \dots, \mathbf{W}_t\big) \ , \ \ t = 0 \dots T-1 \ , \ \ i = 1 \dots N$$

$$\sum_{i=1}^{N} \theta_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i) = 0 \ , \qquad t = 0 \dots T-1$$

Dynamic programming yields centralized controls

- As we want to solve this SOC problem using dynamic programming (**DP**), we suppose to be in the Markovian setting, that is, $\mathbf{W}_0, \dots, \mathbf{W}_T$ are a white noise
- The system is made of N interconnected subsystems, with the control $\mathbf{U}_t^i$ and the state $\mathbf{X}_t^i$ of subsystem i at time t
- The optimal control $\mathbf{U}_t^i$ of subsystem i is a function of the whole system state $\left(\mathbf{X}_t^1, \dots, \mathbf{X}_t^N\right)$

$$\mathbf{U}_t^i = \lambda_t^i\left(\mathbf{X}_t^1, \dots, \mathbf{X}_t^N\right)$$

Naive decomposition should lead to decentralized feedbacks

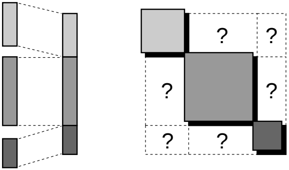
$$\mathbf{U}_t^i = \widehat{\lambda}_t^i(\mathbf{X}_t^i)$$

which are, in most cases, far from being optimal...

Straightforward decomposition of dynamic programming?

The crucial point is that the optimal feedback of a subsystem a priori depends on the state of all other subsystems, so that using a decomposition scheme by subsystems is not obvious...

As far as we have to deal with dynamic programming, the central concern for decomposition/coordination purpose boils down to



- how to decompose a feedback λ_t w.r.t. its domain $\mathbb{X}_t$ rather than its range $\mathbb{U}_t$?

  And the answer is

- impossible in the general case!

Price decomposition and dynamic programming

When applying price decomposition to the problem by dualizing the (almost sure) coupling constraint $\sum_i \theta_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i) = 0$, multipliers $\mathbf{\Lambda}_t^{(k)}$ appear in the subproblems arising at iteration k

$$\min_{\mathbf{U}^i, \mathbf{X}^i} \mathbb{E}\Big[\sum_t L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) + \mathbf{\Lambda}_t^{(k)} \cdot \theta_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i)\Big]$$

- The variables $\mathbf{\Lambda}_t^{(k)}$ are fixed random variables, so that the random process $\mathbf{\Lambda}^{(k)}$ acts as an additional input noise in the subproblems
- But this process may be correlated in time, so that the white noise assumption has no reason to be fulfilled
- DP cannot be applied in a straightforward manner!

Question: how to handle the coordination instruments $\mathbf{\Lambda}_t^{(k)}$ to obtain (an approximation of) the overall optimum?

Outline of the presentation

Decomposition and coordination

A brief insight into scenario decomposition methods

A brief insight into spatial decomposition methods

Summary and research agenda

Let us move to broader stochastic optimization challenges

- Stochastic optimization requires to make risk attitudes explicit
  - robust, worst case, risk measures, in probability, almost surely
- Stochastic dynamic optimization requires to make online information explicit
  - State-based functional approach
  - Scenario-based measurability approach

Numerical walls

- in dynamic programming,
  the bottleneck is the dimension of the state
- in stochastic programming,
  the bottleneck is the number of stages

Here is our research agenda for stochastic decomposition

- Designing risk criteria compatible with decomposition
  - thèse d'Adrien Le Franc (2018—)
- Combining different decomposition methods
  - time: dynamic programming
  - scenario: Progressive Hedging
  - space: decomposition by prices or by quantities
- into blends
  - time + space: Pierre Carpentier talk
    nodal decomposition by prices or by quantities
    + dynamic programming within node
  - time + scenario: Jean-Philippe Chancelier talk
    dynamic programming accross time blocks
    + Progressive Hedging within time blocks