Decomposition/coordination applied to stochastic optimal control problems

Practical aspects and theoretical questions

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Goal of the research

Obtain strategies for a large scale stochastic optimal control problem, for example a problem corresponding to the optimal management over a given time horizon of a system involving a large amount of dynamical production units.

- In order to obtain decision strategies (closed-loop controls), we have to use Dynamic Programming or related methods.
  - **Assumption**: Markovian case,
  - **Difficulty**: curse of dimensionality.
- In order to use Decomposition/Coordination techniques, we have to deal with the information pattern of the stochastic optimization problem.

A long-term effort in our group

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2006 C. Strugarek, "Approches variationnelles et autres contributions en optimisation stochastique", *Thèse de l'ENPC*, mai 2006.

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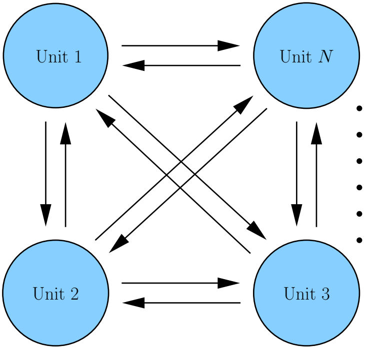
Lecture outline

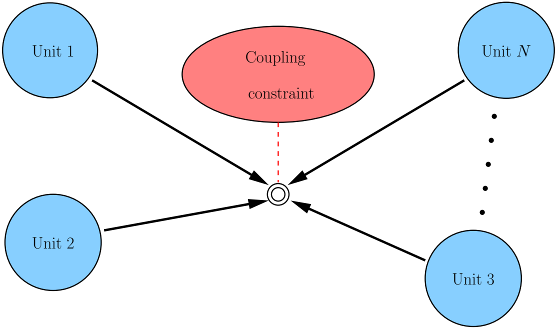
Decomposition and coordination



Interconnected units

- The system to be optimized consists of interconnected subsystems: we want to use this structure in order to formulate optimization subproblems of reasonable complexity.
- But the presence of interactions requires a level of coordination.
- Coordination must provide a local model of the interactions to each subproblem: it is an iterative process.
- The ultimate goal is to obtain the solution of the overall problem by concatenation of the solutions of the subproblems.

Example: the “flower model”



$$\min_{u} \sum_{i=1}^{N} J_i(u_i) \ ,$$

$$\text{s.t.} \quad \sum_{i=1}^{N} \Theta_i(u_i) = \theta \ .$$

Unit Commitment Problem

Price decomposition

$$\min_{u \in \mathcal{U}} \sum_{i=1}^{N} J_i(u_i) \quad \text{subject to} \quad \sum_{i=1}^{N} \Theta_i(u_i) - \theta = 0 \ .$$

1. Form the Lagrangian and assume that a saddle point exists:

$$\max_{\lambda \in \mathcal{V}} \min_{u \in \mathcal{U}} \sum_{i=1}^{N} \Big(J_i(u_i) + \big\langle \lambda \, , \Theta_i(u_i) \big\rangle \Big) - \big\langle \lambda \, , \theta \big\rangle \ .$$

2. Solve this problem by the Uzawa algorithm:

$$u_i^{(k+1)} \in \arg\min_{u_i \in \mathcal{U}_i} J_i(u_i) + \big\langle \lambda^{(k)} \, , \Theta_i(u_i) \big\rangle \ , \quad i = 1 \ldots, N \ .$$

$$\lambda^{(k+1)} = \lambda^{(k)} + \rho \left(\sum_{i=1}^{N} \Theta_i \Big(u_i^{(k+1)} \Big) - \theta \right) .$$

Remarks on decomposition methods

- The theory is available for infinite dimensional Hilbert spaces, and thus applies in the stochastic framework, that is, the case where $\mathcal{U}$ is a space of random variables.
- The minimization algorithm used for solving the subproblems is not specified in the decomposition process.
- New variables appear in the subproblems arising at iteration k of the optimization process:

$$\min_{u_i \in \mathcal{U}_i} J_i(u_i) + \left\langle \lambda^{(k)} , \Theta_i(u_i) \right\rangle .$$

These variables are fixed when solving the subproblems, and do not cause any difficulty, at least in the deterministic case.

Stochastic optimal control (SOC) problem formulation

Consider the following SOC problem:

$$\min_{\mathbf{U},\mathbf{X}} \sum_{i=1}^{N} \left(\mathbb{E}\Big(\sum_{t=0}^{T-1} L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) + K^i(\mathbf{X}_T^i) \Big) \right) ,$$

subject to the constraints:

$$\begin{aligned} \mathbf{X}_0^i &= f_{-1}^i(\mathbf{W}_0) , && i = 1 \dots N , \\ \mathbf{X}_{t+1}^i &= f_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) , \quad && t = 0 \dots T-1 , \ \ i = 1 \dots N , \end{aligned}$$

$$\mathbf{U}_t^i \preceq \mathcal{F}_t := \sigma(\mathbf{W}_0, \dots, \mathbf{W}_t) , \ \ t = 0 \dots T-1 , \ \ i = 1 \dots N ,$$

$$\sum_{i=1}^{N} \Theta_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i) = 0 , \qquad t = 0 \dots T-1 ,$$

Dynamic Programming yields centralized controls

Remember that we want to solve this SOC problem using Dynamic Programming (**DP**) or related methods. We thus assume that we are in the Markovian setting.

The system is made of N interconnected subsystems, and we have denoted the control and the state of subsystem i at time t by $\mathbf{U}_t^i$ and $\mathbf{X}_t^i$ respectively. The optimal control of subsystem i is a function of the whole system state:

$$\mathbf{U}_t^i = \gamma_t^i(\mathbf{X}_t^1, \ldots, \mathbf{X}_t^N) \ .$$

Naive decomposition should lead to decentralized feedbacks:

$$\mathbf{U}_t^i = \widehat{\gamma}_t^i(\mathbf{X}_t^i) \ ,$$

which are, in most cases, far from being optimal...

Optimization problem

The SOC problem under consideration reads:

$$\min_{\mathbf{U},\mathbf{X}} \ \mathbb{E}\left(\sum_{i=1}^{N} \left(\sum_{t=0}^{T-1} L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) + K^i(\mathbf{X}_T^i) \right) \right) , \tag{1a}$$

subject to dynamics constraints:

$$\mathbf{X}_0^i = f_{-1}^i(\mathbf{W}_0) , \tag{1b}$$

$$\mathbf{X}_{t+1}^i = f_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) , \tag{1c}$$

to measurability constraints:

$$\mathbf{U}_t^i \preceq \sigma(\mathbf{W}_0, \ldots, \mathbf{W}_t) , \tag{1d}$$

and to instantaneous coupling constraints

$$\sum_{i=1}^{N} \Theta_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i) = 0 . \qquad \text{Constraints to be } \textbf{dualized} \tag{1e}$$

Assumptions

Assumption 1 (White noise)

Noises $\mathbf{W}_0, \ldots, \mathbf{W}_T$ are independent over time.

Dynamic Programming applies: there is no optimality loss to seek the controls $\mathbf{U}_t^i$ as functions of the state at time t.

Assumption 2 (Constraint qualification)

A saddle point of the Lagrangian $\mathcal{L}$ exists.

$$\mathcal{L}(\mathbf{X}, \mathbf{U}, \mathbf{\Lambda}) = \mathbb{E}\left(\sum_{i=1}^{N} \left(\sum_{t=0}^{T-1} L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) + K^i(\mathbf{X}_T^i) + \sum_{t=0}^{T-1} \mathbf{\Lambda}_t \cdot \Theta_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i) \right) \right) ,$$

the $\mathbf{\Lambda}_t$'s being $\sigma(\mathbf{W}_0, \ldots, \mathbf{W}_t)$-measurable random variables.

Assumption 3 (Uzawa)

Uzawa algorithm applies. . .

Uzawa algorithm

At iteration k of the algorithm,

1. Solve Subproblem i, $i = 1, \ldots, N$, with $\mathbf{\Lambda}^{(k)}$ fixed:

$$\min_{\mathbf{U}^i, \mathbf{X}^i} \mathbb{E}\left(\sum_{t=0}^{T-1} \Big(L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) + \mathbf{\Lambda}_t^{(k)} \cdot \Theta_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i) \Big) + K^i(\mathbf{X}_T^i) \right) ,$$

subject to

$$\mathbf{X}_{t+1}^i = f_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) ,$$
$$\mathbf{U}_t^i \preceq \sigma(\mathbf{W}_0, \ldots, \mathbf{W}_t) ,$$

whose solution is denoted $\left(\mathbf{U}^{i,(k+1)}, \mathbf{X}^{i,(k+1)}\right)$.

2. Update the multipliers $\mathbf{\Lambda}_t$:

$$\mathbf{\Lambda}_t^{(k+1)} = \mathbf{\Lambda}_t^{(k)} + \rho_t \left(\sum_{i=1}^{N} \Theta_t^i\big(\mathbf{X}_t^{i,(k+1)}, \mathbf{U}_t^{i,(k+1)}\big) \right) .$$

Structure of a subproblem

$$\min_{\mathbf{U}^i,\mathbf{X}^i} \mathbb{E}\bigg(\sum_{t=0}^{T-1} \Big(L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) + \mathbf{\Lambda}_t^{(k)} \cdot \Theta_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i) \Big) \bigg) ,$$

subject to

$$\begin{aligned} \mathbf{X}_{t+1}^i &= f_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) , \\ \mathbf{U}_t^i \quad &\preceq \sigma(\mathbf{W}_0, \ldots, \mathbf{W}_t) . \end{aligned}$$

Without some knowledge of the process $\mathbf{\Lambda}^{(k)}$ (we just know that $\mathbf{\Lambda}_t^{(k)} \preceq (\mathbf{W}_0, \ldots, \mathbf{W}_t)$), the informational state of this subproblem at time t cannot be summarized by the physical state $\mathbf{X}_t^i$, and must incorporate all noises prior to time t, that is, $(\mathbf{W}_0, \ldots, \mathbf{W}_t)$.

The state of the subproblem increases with time! Something has to be compressed in order to use Dynamic Programming.

Main idea of DADP

In order to overcome the difficulty induced by the $\boldsymbol{\Lambda}_t^{(k)}$, we choose at each time t a random variable $\mathbf{Y}_t^i$ which is measurable w.r.t. the past noises $\big(\mathbf{W}_0, \dots, \mathbf{W}_t\big)$. We call $\mathbf{Y}^i = \big(\mathbf{Y}_0^i, \dots, \mathbf{Y}_{T-1}^i\big)$ the information process for Subsystem i.

The core idea is to replace the multiplier $\boldsymbol{\Lambda}_t^{(k)}$ at iteration k by its conditional expectation w.r.t. $\mathbf{Y}_t^i$: $\boldsymbol{\Lambda}_t^{(k)} \rightsquigarrow \mathbb{E}\big(\boldsymbol{\Lambda}_t^{(k)} \mid \mathbf{Y}_t^i\big)$.

This will (hopefully) lead to a good approximation if the following condition is met:

$\mathbf{Y}_t^i$ is (sufficiently) correlated to the random variable $\boldsymbol{\Lambda}_t$.

Note that we require that the information process is not influenced by controls.

Subproblem approximation

Using this idea, we replace Subproblem i by:

$$\min_{\mathbf{U}^i,\mathbf{X}^i} \mathbb{E}\left(\sum_{t=0}^{T-1} \left(L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) + \mathbb{E}(\mathbf{\Lambda}_t^{(k)} \mid \mathbf{Y}_t^i) \cdot \Theta_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i) \right) + K^i(\mathbf{X}_T^i) \right) ,$$

subject to

$$\begin{aligned} \mathbf{X}_{t+1}^i &= f_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) , \\ \mathbf{U}_t^i &\preceq \sigma(\mathbf{W}_0, \dots, \mathbf{W}_t) . \end{aligned}$$

The conditional expectation $\mathbb{E}(\mathbf{\Lambda}_t^{(k)} \mid \mathbf{Y}_t^i)$ corresponds to a given function of the variable $\mathbf{Y}_t^i$, so that Subproblem i involves the 2 noises random processes $\mathbf{W}$ and $\mathbf{Y}^i$.

If $\mathbf{Y}^i$ is a short memory process, DP applies effectively.

Dynamic Programming equation

Assuming a non-controlled dynamics for the information process $\mathbf{Y}^i$, the DP equation writes:

$$
\begin{aligned}
V_T^i(x,y) &= K^i(x) \;, \\
V_t^i(x,y) &= \min_u \mathbb{E}\Big(L_t^i(x,u,\mathbf{W}_{t+1}) \\
&\qquad\qquad + \mathbb{E}\big(\mathbf{\Lambda}_t^{(k)} \mid \mathbf{Y}_t^i = y\big) \cdot \Theta_t^i(x,u) \\
&\qquad\qquad\qquad + V_{t+1}^i\big(\mathbf{X}_{t+1}^i, \mathbf{Y}_{t+1}^i\big)\Big) \;,
\end{aligned}
$$

subject to the dynamics:

$$
\begin{aligned}
\mathbf{X}_{t+1}^i &= f_t^i(x,u,\mathbf{W}_{t+1}) \;, \\
\mathbf{Y}_{t+1}^i &= h_t^i(y,\mathbf{W}_{t+1}) \;.
\end{aligned}
$$

Interpretation of DADP

Proposition 1

Assuming that the information process $\mathbf{Y}^i$ *is identical for all subsystems, the DADP algorithm can be interpreted as the Uzawa algorithm applied to the relaxed problem:*

$$\min_{\mathbf{U},\mathbf{X}} \ \mathbb{E}\left(\sum_{i=1}^{N} \left(\sum_{t=0}^{T-1} L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) + K^i(\mathbf{X}_T^i) \right) \right) ,$$

subject to the coupling constraints:

$$\mathbb{E}\left(\sum_{i=1}^{N} \Theta_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i) \;\middle|\; \mathbf{Y}_t \right) = 0 \ .$$

DADP thus consists in replacing an almost-sure constraint by its conditional expectation w.r.t. the information variable $\mathbf{Y}_t$.

$\rightsquigarrow$ *PhD thesis of P. Girardeau.*

Practical questions

⋆ How to choose the information variables $\mathbf{Y}_t^i$?

- Perfect memory: $\mathbf{Y}_t^i = \big(\mathbf{W}_0, \ldots, \mathbf{W}_t\big)$.
- Minimal information: $\mathbf{Y}_t^i \equiv \text{cste}$.
- Static information: $\mathbf{Y}_t^i = h_t^i\big(\mathbf{W}_t\big)$.
- Dynamic information: $\mathbf{Y}_{t+1}^i = h_t^i\big(\mathbf{Y}_t^i, \mathbf{W}_{t+1}\big)$.

⋆ How to obtain a feasible solution from the relaxed problem?

- Use an appropriate heuristic!

⋆ How to accelerate the gradient algorithm?

- Augmented Lagrangian.
- More sophisticated gradient methods.

⋆ How to handle more complex structures than the flower model?

$\rightsquigarrow$ *PhD thesis of J.-C. Alais.*

1. Decomposition and coordination
2. Dual approximate dynamic programming
3. Theoretical questions

What are the issues to consider?

- We treat the coupling constraints in a stochastic optimization problem by duality methods.
- Uzawa algorithm is a dual method which is naturally described in an Hilbert space, but we cannot guarantee the existence of an optimal multiplier in the space $\mathrm{L}^2\big(\Omega, \mathcal{A}, \mathbb{P}; \mathbb{R}^n\big)$!
- Consequently, we extend the algorithm to the non-reflexive Banach space $\mathrm{L}^\infty\big(\Omega, \mathcal{A}, \mathbb{P}; \mathbb{R}^n\big)$, by giving a set of conditions ensuring the existence of a $\mathrm{L}^1\big(\Omega, \mathcal{A}, \mathbb{P}; \mathbb{R}^n\big)$ optimal multiplier, and by providing a convergence result of the algorithm.
- We also have to deal with the approximation induced by the information variable: we give an epi-convergence result related to such an approximation.

Abstract formulation of the problem

We consider the following abstract optimization problem:

$$(\mathcal{P}) \qquad \min_{\mathbf{U}\in\mathcal{U}^{\mathrm{ad}}} J(\mathbf{U}) \quad \text{s.t.} \quad \Theta(\mathbf{U}) \in -C \ ,$$

where $\mathcal{U}$ and $\mathcal{V}$ are two Banach spaces, and

- $J : \mathcal{U} \to \overline{\mathbb{R}}$ is the objective function,
- $\mathcal{U}^{\mathrm{ad}}$ is the admissible set,
- $\Theta : \mathcal{U} \to \mathcal{V}$ is the constraint function, to be dualized,
- $C \subset \mathcal{V}$ is the cone of constraint.

Let $\mathcal{U}^{\Theta} = \big\{\mathbf{U} \in \mathcal{U},\ \Theta(\mathbf{U}) \in -C\big\}$ be the associated constraint set.

Here, $\mathcal{U}$ is a space of random variables, and J is defined by

$$J(\mathbf{U}) = \mathbb{E}\big(j(\mathbf{U}, \mathbf{W})\big) \ .$$

The relationship with Problem (1) is almost straightforward. . .

Standard duality in L^2 spaces

Assume that $\mathcal{U} = \mathrm{L}^2(\Omega, \mathcal{A}, \mathbb{P}; \mathbb{R}^n)$ and $\mathcal{V} = \mathrm{L}^2(\Omega, \mathcal{A}, \mathbb{P}; \mathbb{R}^m)$.

The standard sufficient constraint qualification condition

$$0 \in \mathrm{ri}\Big(\Theta\big(\mathcal{U}^{\mathrm{ad}} \cap \mathrm{dom}(J)\big) + C\Big) \ ,$$

is scarcely satisfied in such a stochastic setting.

Proposition 2

If the σ-algebra $\mathcal{A}$ is not finite modulo $\mathbb{P}$, then for any subset $U^{\mathrm{ad}} \subset \mathbb{R}^n$ that is not an affine subspace, the set

$$\mathcal{U}^{\mathrm{ad}} = \Big\{\mathbf{U} \in \mathrm{L}^p(\Omega, \mathcal{A}, \mathbb{P}; \mathbb{R}^n) \mid \mathbf{U} \in U^{\mathrm{ad}} \quad \mathbb{P} - a.s.\Big\} \ ,$$

has an empty relative interior in L^p, for any $p < +\infty$.

Standard duality in L^2 spaces II

Consider the following optimization problem:

$$
\begin{aligned}
\inf_{u_0,\mathbf{U}_1} \quad & u_0^2 + \mathbb{E}\big((\mathbf{U}_1 + \alpha)^2\big) \;, \\
\text{s.t.} \quad & u_0 \geq a \;, \\
& \mathbf{U}_1 \geq 0 \;, \\
& u_0 - \mathbf{U}_1 \geq \mathbf{W} \;, \qquad \text{to be dualized}
\end{aligned}
$$

where $\mathbf{W}$ is a random variable uniform on $[1,2]$.

For $a < 2$, we can construct a maximizing sequence of multipliers for the dual problem that does not converge in L^2. We are in the so-called *non relatively complete recourse* case, that is, the case where the constraints on $\mathbf{U}_1$ induce a stronger constraint on u_0.

An optimal multiplier is available in $(\mathrm{L}^\infty)^\star$...

Constraint qualification in $\left(\mathrm{L}^{\infty}, \mathrm{L}^{1}\right)$

From now on, we assume that

$$\mathcal{U} = \mathrm{L}^{\infty}\left(\Omega, \mathcal{A}, \mathbb{P}; \mathbb{R}^{n}\right) ,$$
$$\mathcal{V} = \mathrm{L}^{\infty}\left(\Omega, \mathcal{A}, \mathbb{P}; \mathbb{R}^{m}\right) ,$$
$$C = \{0\} ,$$

where the σ-algebra $\mathcal{A}$ is not finite modulo $\mathbb{P}$.[2]

We consider the pairing $\left(\mathrm{L}^{\infty}, \mathrm{L}^{1}\right)$ with the following topologies:

- $\sigma\left(\mathrm{L}^{\infty}, \mathrm{L}^{1}\right)$: weak* topology on L^{∞} (coarsest topology such that all the L^{1}-linear forms are continuous),
- $\tau\left(\mathrm{L}^{\infty}, \mathrm{L}^{1}\right)$: Mackey-topology on L^{∞} (finest topology such that the continuous linear forms are only the L^{1}-linear forms).

[2] If the σ-algebra is finite modulo $\mathbb{P}$, $\mathcal{U}$ and $\mathcal{V}$ are finite dimensional spaces.

Weak* closedness of linear subspaces of L^∞

Proposition 3

Let $\Theta : \mathrm{L}^\infty(\Omega, \mathcal{A}, \mathbb{P}; \mathbb{R}^n) \to \mathrm{L}^\infty(\Omega, \mathcal{A}, \mathbb{P}; \mathbb{R}^m)$ *be a linear operator, and assume that there exists a linear operator* $\Theta^\dagger : \mathrm{L}^1(\Omega, \mathcal{A}, \mathbb{P}; \mathbb{R}^m) \to \mathrm{L}^1(\Omega, \mathcal{A}, \mathbb{P}; \mathbb{R}^n)$ *such that:*

$$\langle \mathbf{V} , \Theta(\mathbf{U}) \rangle = \langle \Theta^\dagger(\mathbf{V}) , \mathbf{U} \rangle \quad \forall \mathbf{U},\ \forall \mathbf{V} .$$

Then the linear operator Θ *is weak** *continuous.*

Applications

- $\Theta(\mathbf{U}) = \mathbf{U} - \mathbb{E}\big(\mathbf{U} \bigm| \mathcal{B}\big)$: non-anticipativity constraints,
- $\Theta(\mathbf{U}) = A\mathbf{U}$ with $A \in \mathcal{M}_{m,n}(\mathbb{R})$: finite number of constraints.

A duality theorem

$$(\mathcal{P}) \qquad \min_{\mathbf{U}\in\mathcal{U}} J(\mathbf{U}) \quad \text{s.t.} \quad \Theta(\mathbf{U}) = 0 \; ,$$

with $J(\mathbf{U}) = \mathbb{E}\big(j(\mathbf{U},\mathbf{W})\big)$.

Theorem 1

Assume that j is a convex normal integrand, that J is continuous in the Mackey topology at some point $\mathbf{U}_0$ such that $\Theta(\mathbf{U}_0) = 0$, and that Θ is weak* continuous on $\mathrm{L}^\infty\big(\Omega,\mathcal{A},\mathbb{P};\mathbb{R}^n\big)$.
Then, $\mathbf{U}^\sharp \in \mathcal{U}$ is an optimal solution of Problem $\big(\mathcal{P}\big)$ if and only if there exists $\boldsymbol{\lambda}^\sharp \in \mathrm{L}^1\big(\Omega,\mathcal{A},\mathbb{P};\mathbb{R}^m\big)$ such that

- $\mathbf{U}^\sharp \in \arg\min\limits_{\mathbf{U}\in\mathcal{U}} \mathbb{E}\Big(j(\mathbf{U},\mathbf{W}) + \boldsymbol{\lambda}^\sharp \cdot \Theta(\mathbf{U})\Big)$,
- $\Theta(\mathbf{U}^\sharp) = 0$.

Extension of a result given by R. Wets for non-anticipativity constraints.

Uzawa algorithm

$$(\mathcal{P}) \qquad \min_{\mathbf{U}\in\mathcal{U}} J(\mathbf{U}) \quad \text{s.t.} \quad \Theta(\mathbf{U}) = 0 \; .$$

with $J(\mathbf{U}) = \mathbb{E}\big(j(\mathbf{U},\mathbf{W})\big)$.

The standard Uzawa algorithm

$$\mathbf{U}^{(k+1)} \in \operatorname*{arg\,min}_{\mathbf{U}\in\mathcal{U}^{\mathrm{ad}}} \; J(\mathbf{U}) + \big\langle \boldsymbol{\lambda}^{(k)} \,, \Theta(\mathbf{U}) \big\rangle \; ,$$
$$\boldsymbol{\lambda}^{(k+1)} = \boldsymbol{\lambda}^{(k)} + \rho \, \Theta\big(\mathbf{U}^{(k+1)}\big) \; ,$$

makes sense with in the L^∞ setting, that is, the minimization problem is well-posed and the update formula is valid one.

Note that all the multipliers $\boldsymbol{\lambda}^{(k)}$ belong to $\mathrm{L}^\infty\big(\Omega, \mathcal{A}, \mathbb{P}; \mathbb{R}^m\big)$ as soon as the initial multiplier $\boldsymbol{\lambda}^{(0)} \in \mathrm{L}^\infty\big(\Omega, \mathcal{A}, \mathbb{P}; \mathbb{R}^m\big)$.

Convergence result

Theorem 2

Assume that

1. $J : \mathcal{U} \to \overline{\mathbb{R}}$ *is proper, weak** *l.s.c., differentiable and* a*-convex,*
2. $\Theta : \mathcal{U} \to \mathcal{V}$ *is affine, weak** *continuous and* κ*-Lipschitz,*
3. $\mathcal{U}^{\mathrm{ad}}$ *is weak** *closed and convex,*
4. *an admissible* $\mathbf{U}_0 \in \mathrm{dom}\, J \cap \Theta^{-1}(0) \cap \mathcal{U}^{\mathrm{ad}}$ *exists,*
5. *an optimal* L^1*-multiplier to the constraint* $\Theta\big(\mathbf{U}\big) = 0$ *exists,*
6. *the step* ρ *is such that* $0 < \rho < \frac{2a}{\kappa}$.

Then, there exists a subsequence $\big\{\mathbf{U}^{(n_k)}\big\}_{k\in\mathbb{N}}$ *of the sequence generated by the Uzawa algorithm converging in* L^∞ *toward the optimal solution* $\mathbf{U}^\sharp$ *of the primal problem.*

Remarks about these results

- The result is not as good as expected (global convergence).
- Improvements and extensions (inequality constraint) needed.
- The Mackey-continuity assumption forbids the use of bounds.
  - In order to deal with almost sure bound constraints, we can turn towards the work of R.T. Rockafellar and R. J-B Wets.
  - In a series of 4 papers (stochastic convex programming), they have detailed the duality theory on two-stage and multistage problems, with the focus on non-anticipativity constraints.
  - These papers require:
    - a strict feasability assumption,
    - a relatively complete recourse assumption.

Conclusion

- DADP method allows to tackle large-scale stochastic optimal control problems, such as the ones found in the field of energy management.
- A lot of practical experiments have been performed, both on "flower models" (Unit Commitment Problem) and "chained models" (hydraulic valley management), and much work remains to be done in this area.
- A more theoretical work has begun in order to provide the foundations of the method.

More information: PhD defense of Vincent Leclère, Wednesday, June 25 at École des Ponts ParisTech.

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