TUNED HARMONIC ABSORBERS FOR ENGINE TORSIONAL VIBRATIONS

PSA GROUPE

RESEARCH & DEVELOPMENT

DRD/DRIA/DSTF

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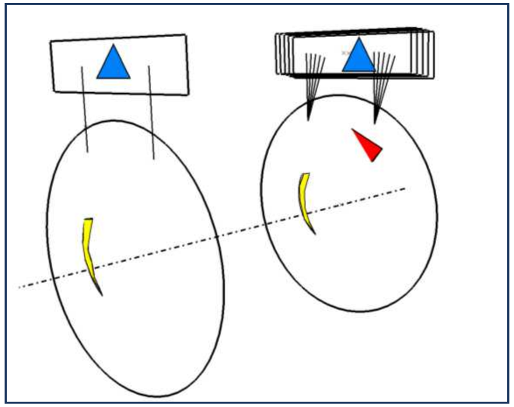
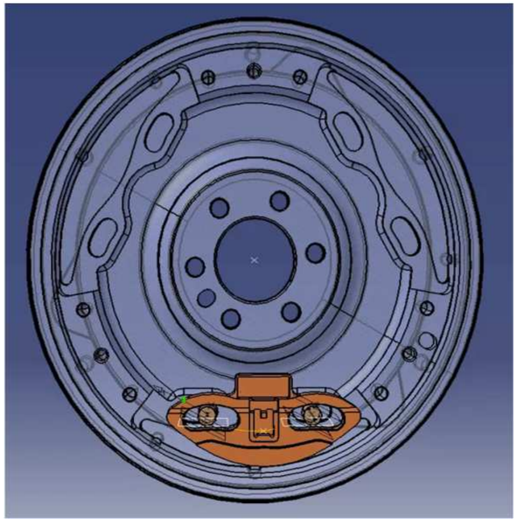
GDR DYNOLIN

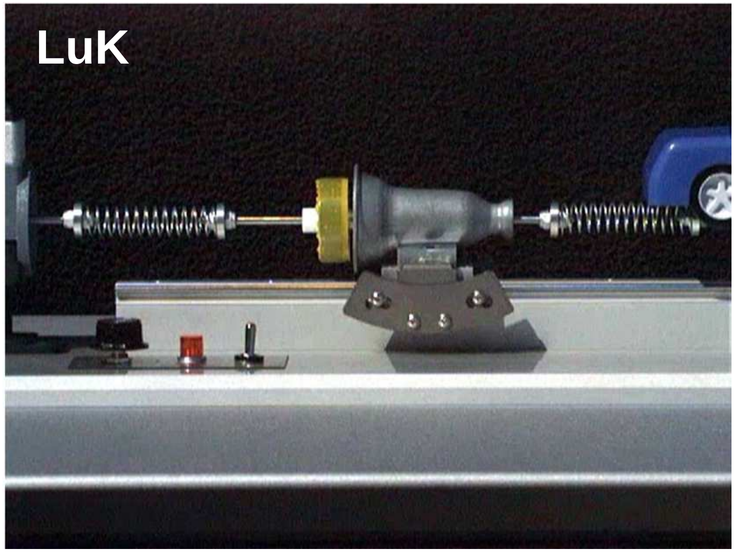
Zoran DIMITRIJEVIC

CONTEXT AND ISSUES

- Torsional vibrations absorption using pendular sytems



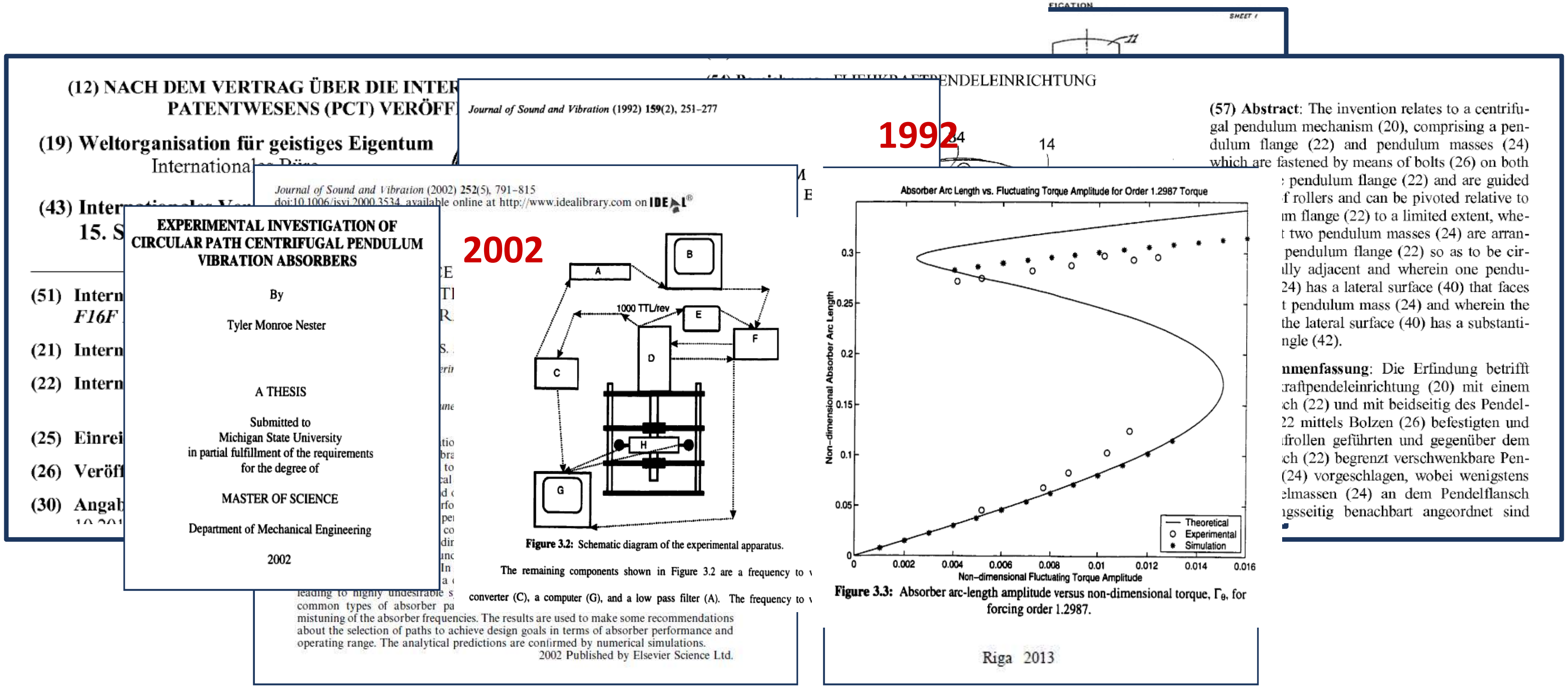




- Limiting harmonics on clutches, flywheels, cankshafts, camshafts :
    - H2 on 4 cylinders engine.
    - H1,5 and H3 on 3 cylinders engine.
    - Here we want to prevent h0,75 on a 3 cylinders engine.

CONTEXT AND ISSUES

- History and bibliography
  - Since the early 1910's (Kutzbach 1911) multiple applications of the oscillating pendular system for damping torsional vibrationts:

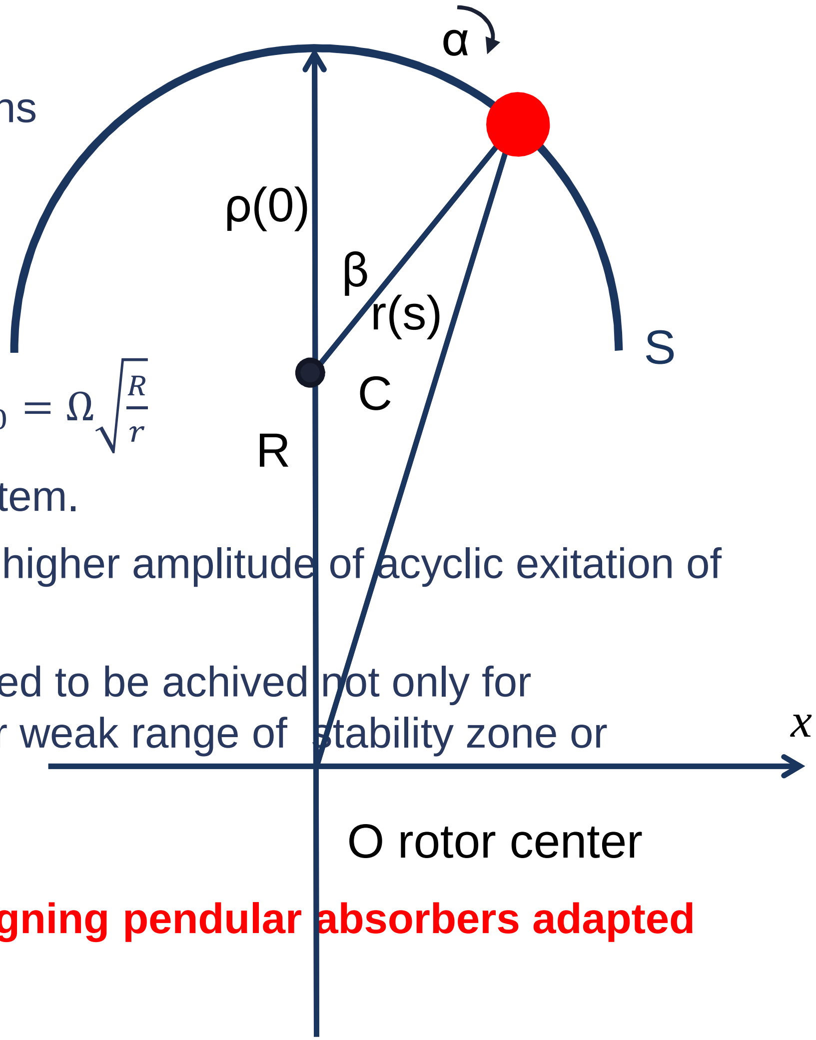


CONTEXT AND ISSUES

- Using « classical » linearized Lagrangian equations

$$[I + J + m(R+r)^2]\ddot{\alpha} + [mr(R+r)]\ddot{\beta} = \gamma$$
$$mr(R+r)\ddot{\alpha} + mr^2\ddot{\beta} + mRr\Omega^2\beta = 0$$

- Then the absorbers are tuned to the frequency $\Omega_0 = \Omega\sqrt{\frac{R}{r}}$
- This is true only for weak perturbations of the system.
- In the industrial context we need robust paths for higher amplitude of acyclic exitation of the rotor.
- The nonlinear analyzes of the coupled system need to be achived not only for epicycloidal or tautochrone paths because of their weak range of stability zone or efficiency.



- **The main aim is to produce a tool for pre-designing pendular absorbers adapted on the flywheel of a 3 cylinders engine**.

NONLINEAR DYNAMIC EQUATIONS OF THE COUPLED SYSTEM ENGINE FLYWHEEL WITH PENDULAR ABSORBERS

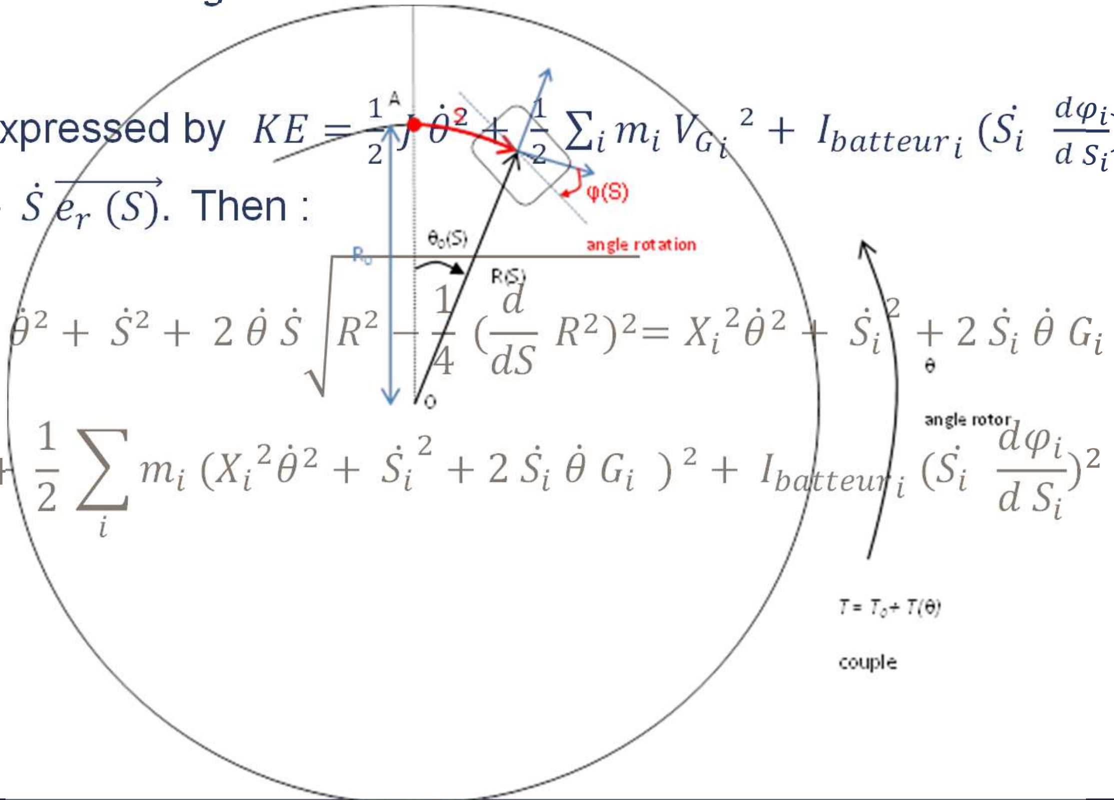
- We try to find the steady responses in 1:1 resonance of the coupled system rotor with N pendulars when pendulars are in a synchronous motion.

- The pendulars can rotate, and the path is represented by the distance from the center of rotation, function of an arclength variable S.

- Kinetic energy is expressed by $KE = \frac{1}{2} J\dot{\theta}^2 + \frac{1}{2}\sum_i m_i {V_{G_i}}^2 + I_{batteur_i}(\dot{S_i}\ \frac{d\varphi_i}{d\,S_i})^2$ with $\overrightarrow{V_G} = R(S)\dot{\theta}\ \overrightarrow{e_\theta(S)} + \dot{S}\ \overrightarrow{e_r\ (S)}$. Then :

$$ {V_G}^2 = R^2\,\dot{\theta}^2 + \dot{S}^2 + 2\,\dot{\theta}\,\dot{S}\sqrt{R^2 - \frac{1}{4}\left(\frac{d}{dS}R^2\right)^2} = {X_i}^2\dot{\theta}^2 + {\dot{S}_i}^2 + 2\,\dot{S}_i\,\dot{\theta}\,G_i $$

$$ KE = \frac{1}{2}J\,\dot{\theta}^2 + \frac{1}{2}\sum_i m_i\,({X_i}^2\dot{\theta}^2 + {\dot{S}_i}^2 + 2\,\dot{S}_i\,\dot{\theta}\,G_i\ )^2 + I_{batteur_i}(\dot{S_i}\ \frac{d\varphi_i}{d\,S_i})^2 $$



NONLINEAR DYNAMIC EQUATIONS OF THE COUPLED SYSTEM ENGINE FLYWHEEL WITH PENDULAR ABSORBERS

- Lagrangian equations of motion for the coupled system are written :

$$J\ddot{\theta} + \sum_j m_j \left\{ \frac{dX_j}{dS_j} \dot{S_j} \ \dot{\theta} + X_j(S_j)\ddot{\theta} + G_j(S_j)\ddot{S_j} + \frac{dG_j}{dS_j} \dot{S_j}^2 \right\} + \ I_{batteur\,j} \left(\frac{d\varphi_j}{dS_j} \ddot{S_j} + \frac{d^2\varphi_j}{dS_j^2} \dot{S_j}^2 \right) =$$
$$\sum_j c_{a\,j}\, G_j(S_j)\dot{S_j} \ - \ c_0\, \dot{\theta} + \ T_0 + T(\theta)$$

$$-\frac{1}{2}\, m_i\, \frac{d\, X_i}{S_i}\, \dot{\theta}^2 + \ m_i\, (\ddot{S_i} + G_i(S_i)\ddot{\theta}) + \ I_{batteur_i} \left(\left(\frac{d\, \varphi_i}{d\, S_i} \right)^2 \ddot{S_i} + \frac{d\, \varphi_i}{d\, S_i}\, S_i\, \ddot{\theta} \right) + c_{a_i}\, S_i \dot{=} 0$$

- Adimensionnalization using $s_i = \frac{S_i}{R_i(0)}$, $X_i(S_i) = R_i(0)^2 x(s_i)$, $G(S_i) = R_i(0) g(s_i)$ is done:

$$\ddot{\theta} + \sum_j b_j \left\{ \frac{dx_j}{ds_j} \dot{s_j} \ \dot{\theta} + x_j(s_j)\ddot{\theta} + g_j(s_j)\ddot{s_j} + \frac{dg_j}{ds_j} \dot{s_j}^2 \right\} \ + b_j\, K_j \left\{ \frac{d\varphi_j}{d\, s_j} \ddot{s_j} + \frac{d^2\varphi_j}{dS_j^2} \dot{s_j}^2 \right\}$$
$$= \sum_j \frac{c_{a\,j}\, R_j(0)^2}{J} \ \ g_j(s_j)\dot{s_j} \ - \frac{c_0}{J}\, \dot{\theta} + \frac{T_0}{J} + \frac{T(\theta)}{J}$$

$$-\frac{1}{2}\, b_i\, \frac{dx_i}{ds_i}\, \dot{\theta}^2 + \ b_i \left\{ \ddot{s_i} + g_i(s_i)\ddot{\theta} \right\} \ + \ b_i\, K_i \left\{ \left(\frac{d\varphi_i}{ds_i} \right)^2 \ddot{s_i} + \frac{d\varphi_i}{ds_i}\, s_i\, \ddot{\theta} \right\} = \ - \ \frac{c_{a_i}\, R_i(0)^2}{J} \dot{s_i}$$

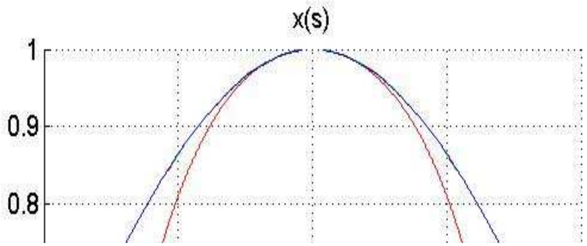
With $b_i = \frac{m_i R_i(0)^2}{J}$, $I_{batteur_i} = m_i\, \rho_i^2$, $K_i = \left(\frac{\rho_i}{R_i(0)} \right)^2$

NONLINEAR DYNAMIC EQUATIONS OF THE COUPLED SYSTEM ENGINE FLYWHEEL AND PENDULAR ABSORBERS

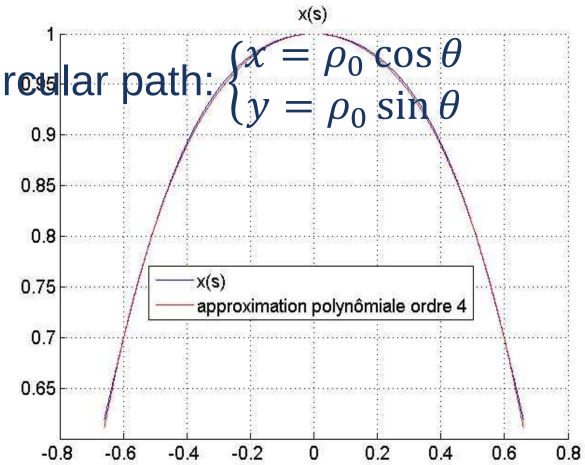
- Approximations of the path curvature:
  - $x(s) = 1 - \tilde{n}^2 s^2 + \gamma s^4 + o(s^4)$
  - $g(s) = 1 - \frac{1}{2}(\tilde{n}^2 + \tilde{n}^4)s^2 + \frac{1}{4}\left(2\gamma(1 + 4\tilde{n}^2) + \frac{1}{2}(\tilde{n}^2 + \tilde{n}^4)^2\right)s^4 + o(s^4)$
  - $\varphi(s) = \alpha\, s + \beta\, s^2 + \delta\, s^3 + o(s^3)$

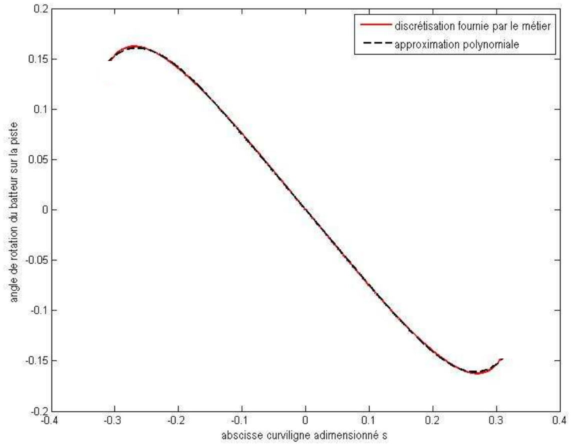
"Hardening" path

$$\begin{cases} x = b(1 + k_1\theta^2 + k_2\theta^4)\cos\theta \\ y = b(1 + k_1\theta^2 + k_2\theta^4)\sin\theta \end{cases}$$



Circular path: $\begin{cases} x = \rho_0 \cos\theta \\ y = \rho_0 \sin\theta \end{cases}$





NONLINEAR DYNAMIC EQUATIONS OF THE COUPLED SYSTEM ENGINE FLYWHEEL AND PENDULAR ABSORBERS

- We define the scale factor $\varepsilon = \frac{N\, m_i\, R_i(0)^2}{J}$
- Considering the next variable changes and introducing the detuning parameter σ:

$$\begin{cases} v = \frac{\dot{\theta}}{\Omega} \\ \ddot{\theta} = \Omega^2 v v' \\ (\dot{\ }) = \Omega v (\ )' \\ (\ddot{\ }) = \Omega^2 v v' (\ )' + \Omega^2 v^2 (\ )'' \end{cases}$$

  - $v = 1 + \varepsilon^{3/2} w + o\left(\varepsilon^{3/2}\right)$
  - $s_i = \sqrt{\varepsilon}\, z_i,\ \ \widetilde{\mu_i} = \varepsilon\, \mu_i,\ \tilde{\Gamma} = \varepsilon^{3/2}\, \Gamma,\ \ \tilde{n} = n(1 + \varepsilon\sigma)$
- The motion equations become:

$$\boldsymbol{v' = \varepsilon^{3/2} \left\{ \tilde{\Gamma} + \frac{n^2}{N} \sum_j \frac{z_j}{(1 + K_j\, \alpha_j^2)} \right\}}$$

$$\boldsymbol{z_i''\left(1 + K_i\, \alpha_i^2\right) + n^2 z_i = \sqrt{\varepsilon}\{-4\, K_i\, \alpha_i \beta_i\, z_i\, z_i''\} + \varepsilon \left\{ 2\, \gamma\, z_i{}^3 - \widetilde{\mu}_{a,i}\, z_i' - \tilde{\Gamma} - 2\, n^2 \sigma z_i - 4\, K_i\, \beta_i^2\, z_i^2 z_i'' - 6\, K_i\, \alpha_i\, \delta_i\, z_i^2 z_i'' - \frac{n^2}{N} \sum_j \frac{z_j}{(1 + K_j\, \alpha_j^2)} \right\}}$$

NONLINEAR DYNAMIC EQUATIONS OF THE COUPLED SYSTEM ENGINE FLYWHEEL AND PENDULAR ABSORBERS

- By the hypothesis of a weak nonlinearity, we look for solutions of the type $z_i = a_i \sin(\omega_{0_i}\theta + \psi_i)$. The motion of the pendular absorbers are then solution of:

$$\begin{pmatrix} a_i' \\ a_i\,\varphi_i' \end{pmatrix} = \begin{pmatrix} \sin(\omega_{0_i}\theta + \psi_i) & \frac{1}{A}\cos(\omega_{0_i}\theta + \psi_i) \\ \cos(\omega_{0_i}\theta + \psi_i) & -\frac{1}{A}\sin(\omega_{0_i}\theta + \psi_i) \end{pmatrix} \begin{pmatrix} 0 \\ \sqrt{\varepsilon}\, f_1 + \varepsilon\, g_1 \end{pmatrix}$$

- The averaged equations are then:

$$\begin{pmatrix} \bar a_i' \\ \bar a_i \bar\psi_i' \end{pmatrix} = \begin{pmatrix} \int_0^{\frac{2\pi}{\omega_{0_i}}} \frac{\cos(\omega_{0_i}\theta + \bar\psi_i)}{\omega_{0_i}} \left(\varepsilon\, \overline{g_1} + \varepsilon\, \overline{f_1\, h_1}\right) d\theta \\ \int_0^{\frac{2\pi}{\omega_{0_i}}} -\frac{\sin(\omega_{0_i}\theta + \bar{\bar\psi}_i)}{\omega_{0_i}} \left(\varepsilon\, \overline{g_1} + \varepsilon\, \bar f_1\, \overline{h_1}\right) d\theta \end{pmatrix}$$

- Synchronous steady responses are solved for $\bar a_i' = 0$ and $\bar\varphi_i' = 0$. Let be $\bar a_i = \bar a_j = r,\ \bar\psi_i = \bar\psi_j = \psi$, $\tilde\mu_{a,i} = \tilde\mu_{a,j} = \tilde\mu,\ \omega_{0_i} = \omega_{0_j} = \omega_0$, we finally solve (r,ψ):

$$r\,\pi\,\tilde\mu = \frac{\tilde\Gamma}{(n^2 - \omega_0^{\,2})}\left\{-2\,n\cos(\varphi)\,\sin^2\left(\frac{n\,\pi}{\omega_0}\right) + \omega_0\sin(\varphi)\sin\left(\frac{2\,\pi\,n}{\omega_0}\right)\right\}$$

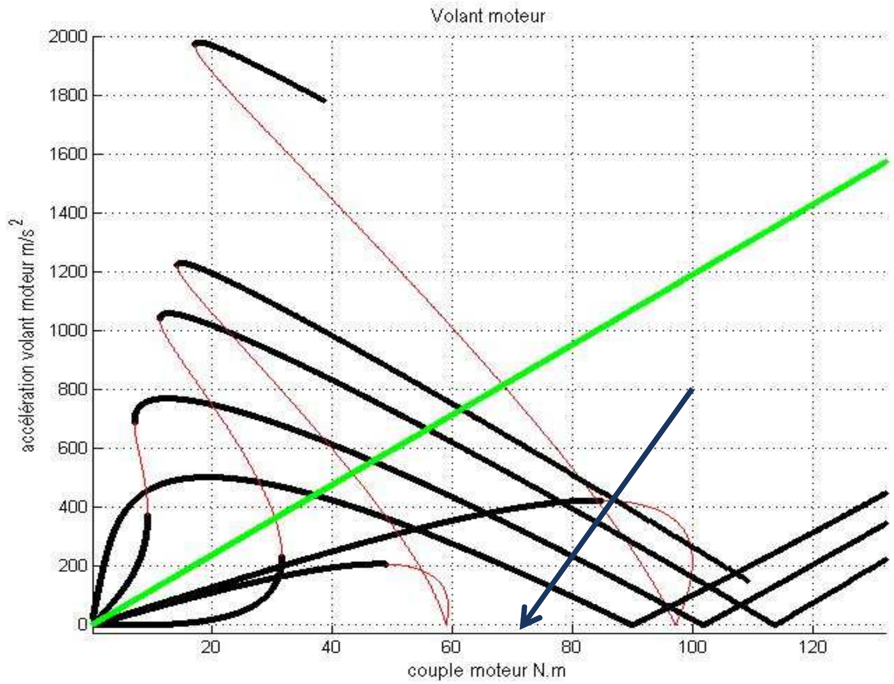
$$\frac{\pi}{2\,\omega_0}\left[-4r\left(1 + K\alpha^2\right)\sigma\,\omega_0^2 + 3r^3\left[\gamma + K(2\,\beta^2 + 3\,\alpha\,\delta)\omega_0^2\right] - 2\,\omega_0^2\, r\right] = \frac{\tilde\Gamma}{(n^2 - \omega_0^{\,2})}\left\{-2\,n\sin(\psi)\sin^2\left(\frac{n\,\pi}{\omega_0}\right) + \omega_0\cos(\psi)\sin\left(\frac{2\,\pi\,n}{\omega_0}\right)\right\}$$

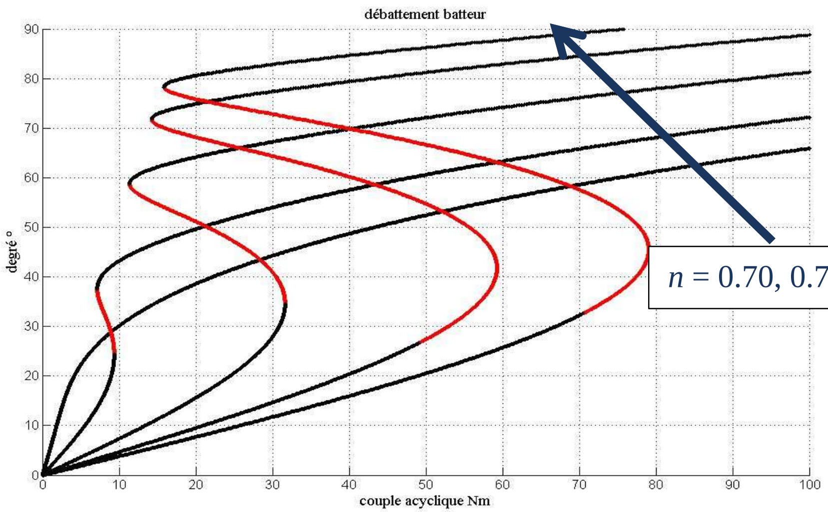
- We can obtain acyclic torque expression of the flywheel using (r,ψ):

$$\tilde\Gamma^2 = \frac{(\omega_0^{\,2} - n^2)^2}{4\,n^2\sin^2\left(\frac{n\,\pi}{\omega_0}\right) + \omega_0^2\sin^2\left(\frac{2\,n\,\pi}{\omega_0}\right)}\left\{(r\,\pi\tilde\mu)^2 + \frac{\pi^2}{4\,\omega_0^{\,2}}\left[-4r\left(1 + K\alpha^2\right)\omega_0^2 + 3r^3\left[\gamma + K(2\,\beta^2 + 3\,\alpha\,\delta)\omega_0^2\right] - 2\,\omega_0^2\, r\right]^2\right\}$$

DESIGN OF THE HARMONIC ABSORBERS

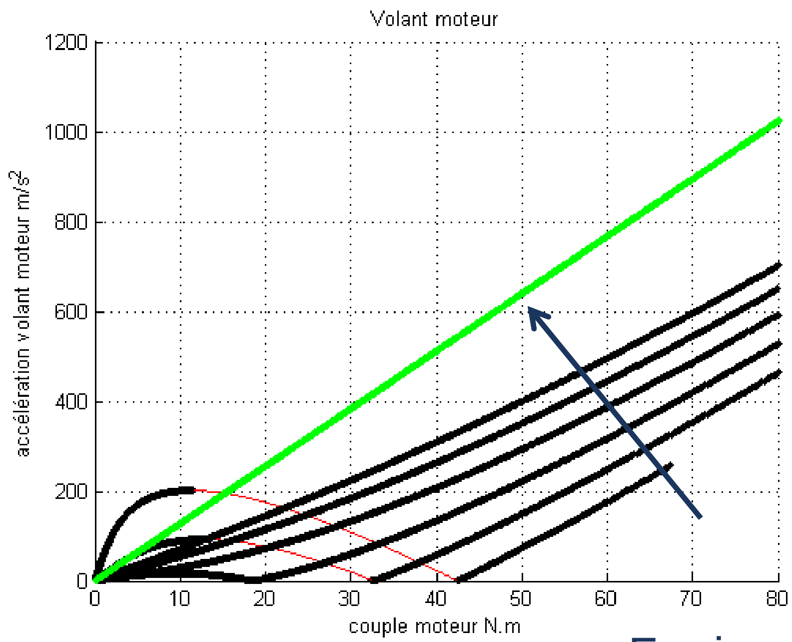
- Recall for circular paths

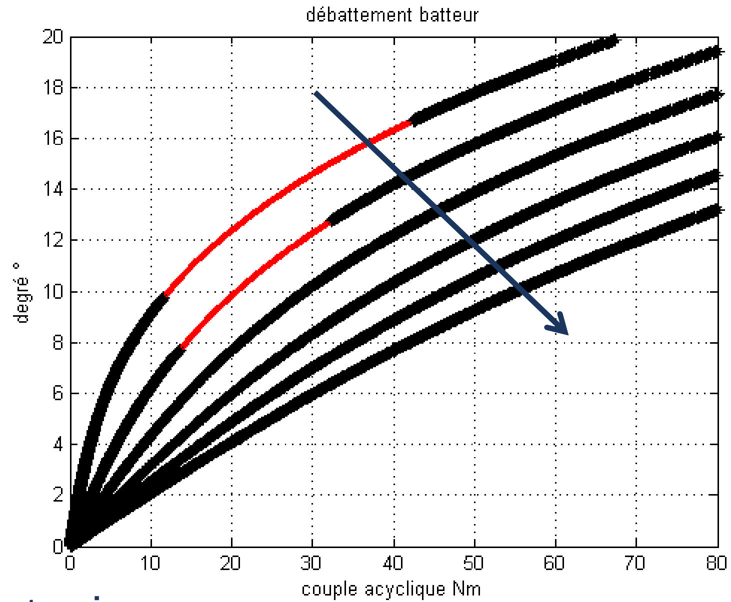




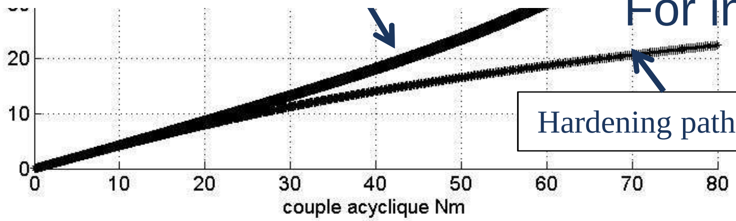
n = 0.70, 0.72, 0.75, 0.78, 0.80

DESIGN OF THE HARMONIC ABSORBERS

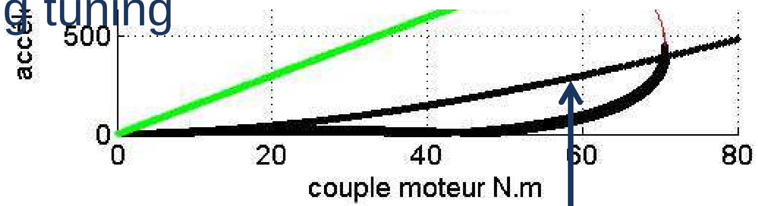




For increasing tuning

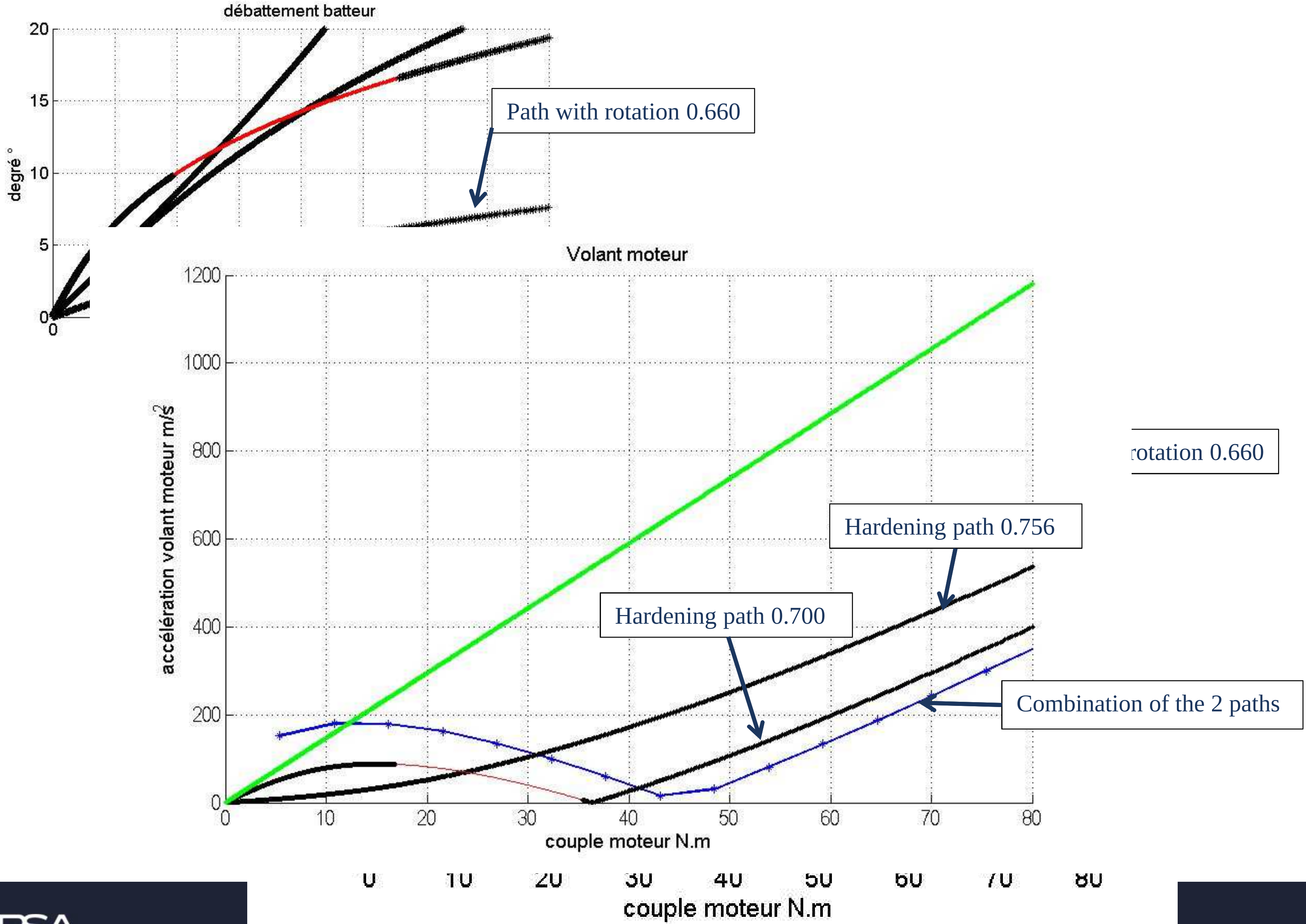


Hardening path 0.756



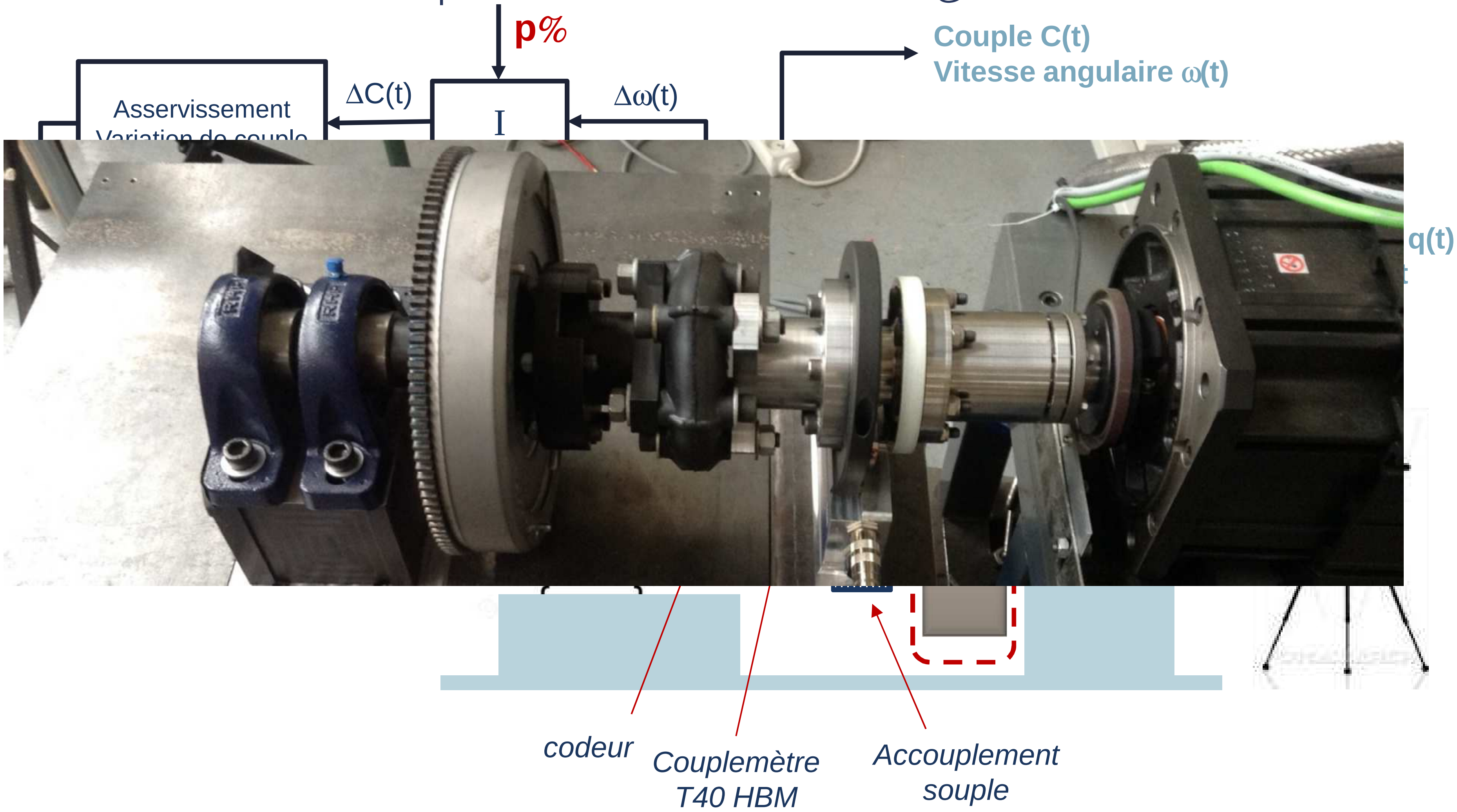
Hardening path 0.756

DESIGN OF THE HARMONIC ABSORBERS



ORGANIC BENCH VALIDATION OF THE CONCEPT

- A test bench was develloped in collaboration with ENSAM @ Aix en Provence:



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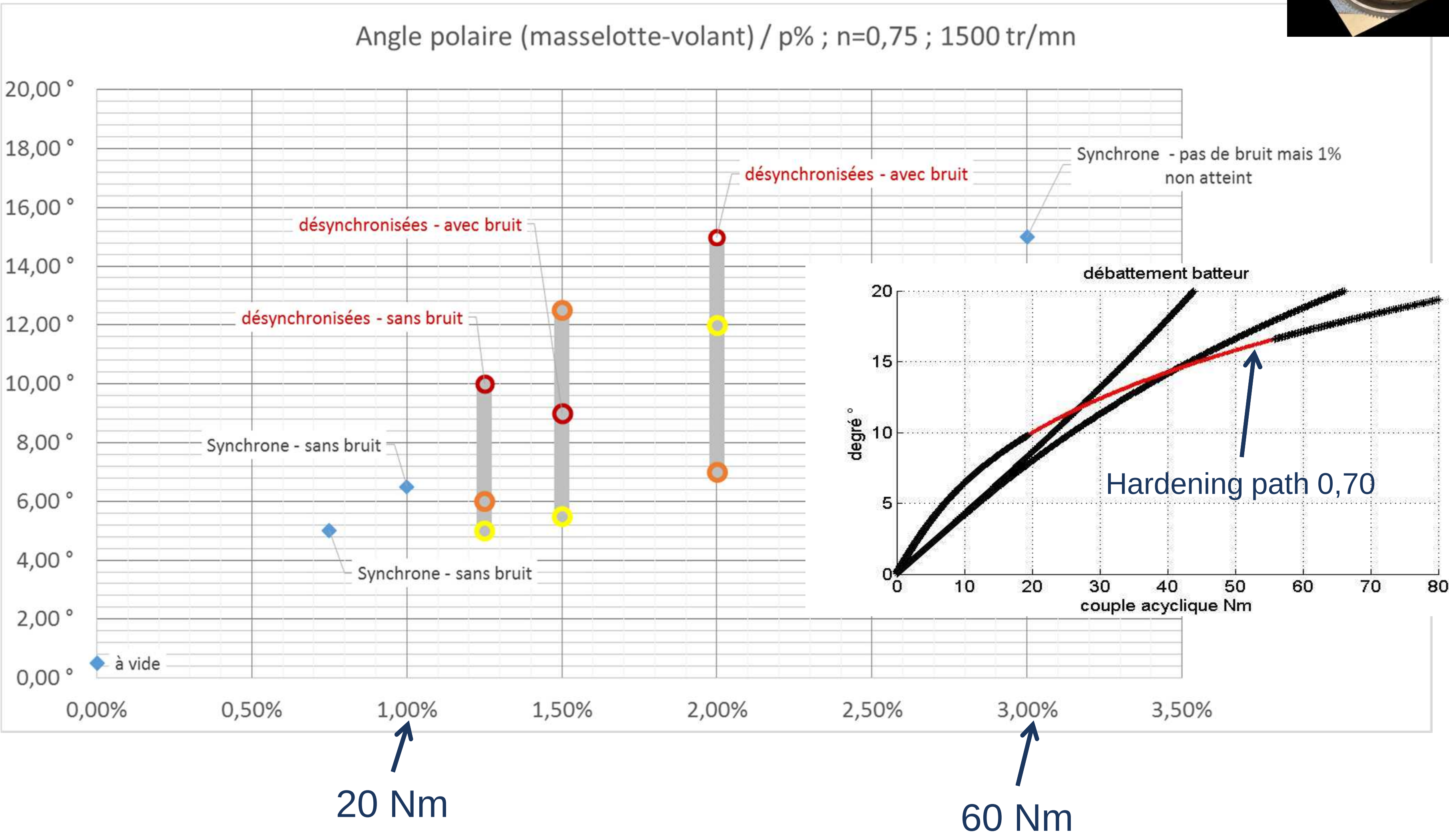
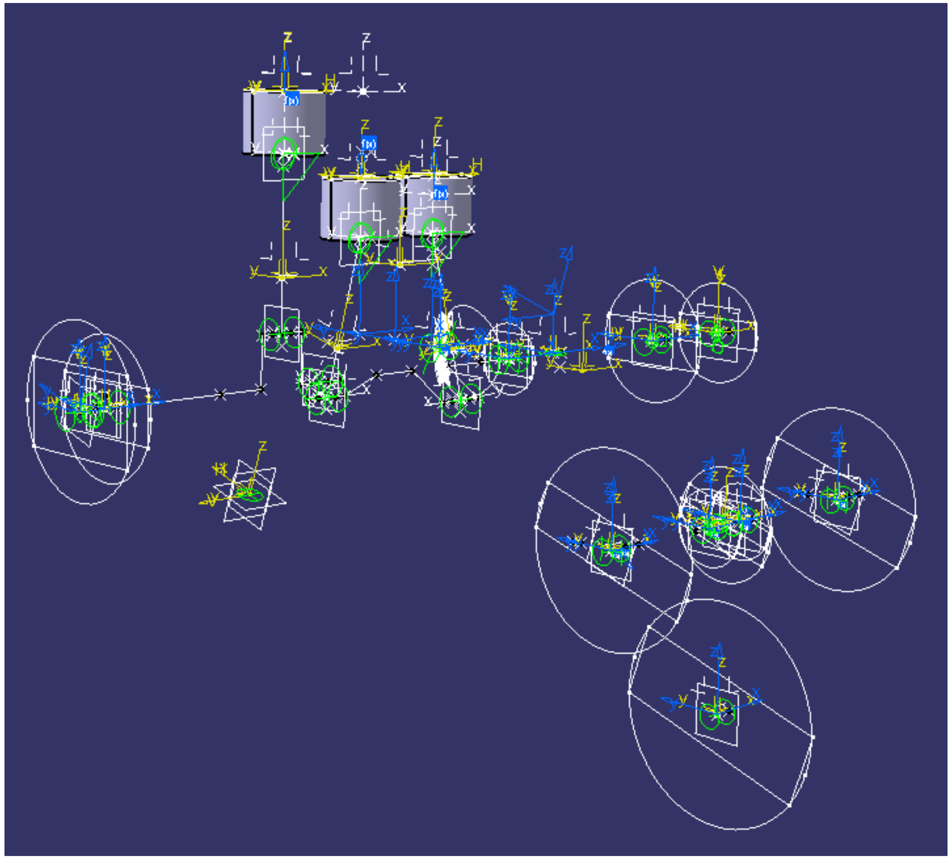
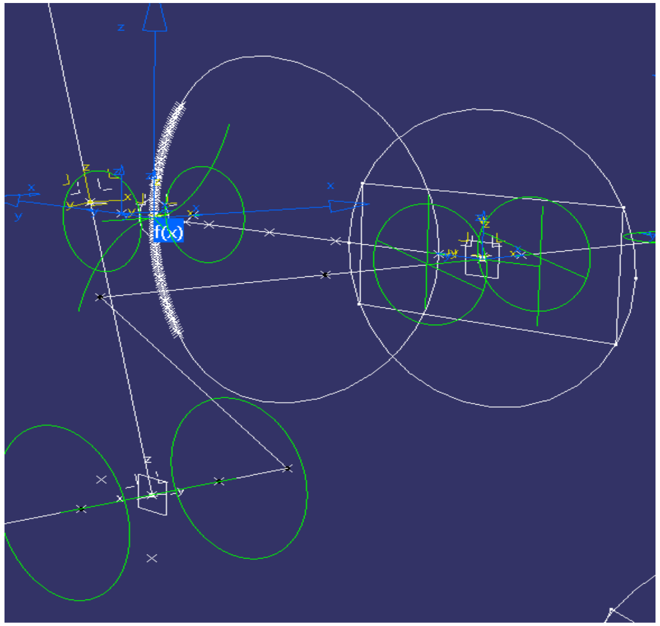


SIMULATION OF THE POWERTRAIN WITH PENDULAR ABSORBERS

- Further simulations integrated in a vehicle powertrain synthesis model





ROTOR WITH PENDULARS TUNED TO H1,5 AND H0.075

- Here we need to consider two acyclic excitation of the rotor

$$T(\theta) = T_0 + T_n sin(n\theta) + T_{2n} sin(2n\theta + \phi)$$

- Dynamics equations are not averaged and we use MANLAB 2.0 to explore branches of steady solutions of the system.

$$z_i'' + n^2 z_i = \varepsilon \left(2\gamma_0 - 2n_i^2 - 2n_i^2 \sigma_i z_i - \tilde{\mu}_{ai} z_i' - \frac{1}{N} \sum_{j=1}^{N} n_i^2 z_i - \widetilde{\Gamma_n} \sin(n\theta) - \widetilde{\Gamma_{2n}} \sin(2n\theta + \phi) \right)$$

$$n = n_i = \tilde{n_i} + \varepsilon \sigma_i \quad i = 1 \ldots N_1$$

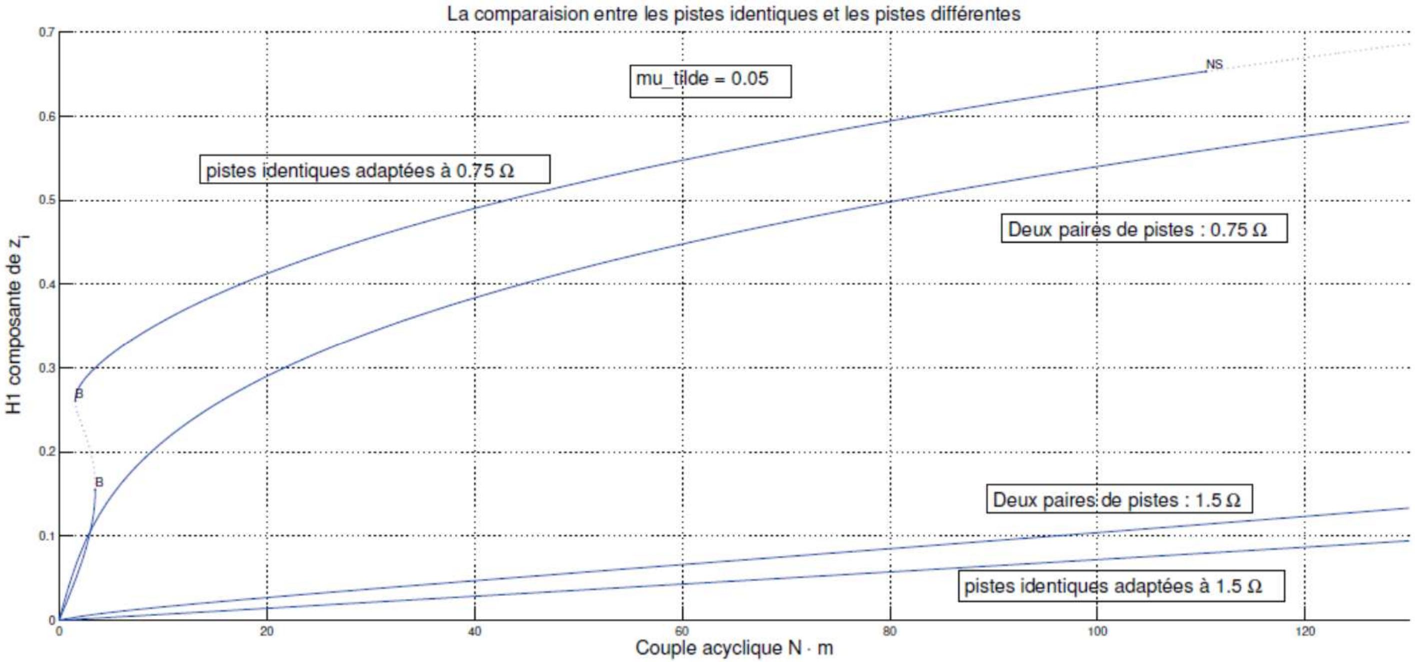
$$2n = n_i = \tilde{n_i} + \varepsilon \sigma_i \quad i = N_1 + 1 \ldots N$$

$$\sum_{i=1}^{N} m_i = m, \qquad \lambda m = \sum_{i=1}^{N_1} m_i\,, \qquad (\lambda - 1)m = \sum_{i=N_1+1}^{N} m_i$$

- The aim is
  - To understand the coupling between pendulars and possible multiple resonances,
  - To give the efficient inertia partition λ for the two paths considered.

ROTOR WITH PENDULARS TUNED TO H1.5 AND H0.75

- First results



CONCLUSION AND PERSPECTIVES

- Hardening paths with rotations seems to be convenient for the automotive applications for low harmonics dissipation…

- The analytical and numerical simulations are validated by the first measures on a organic bench

- We need :
  - To explore asynchronous motion of the pendular absorbers : following the branches of solutions after bifurcations, determine internal resonances seen on the bench…
  - To take into account mechanical stop effect on the dynamics;
  - To further explore the adaption of 2 different tuning paths on the rotor
- We should use more complex absorbers creating a metamaterial composed of different paths adapted to n, 2n, 3n harmonics…