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COMPLICATED DYNAMICS EXHIBITED BY THIN SHELLS DISPLAYING NUMEROUS INTERNAL RESONANCES : APPLICATION TO THE STEELPAN

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Nonlinear vibrations of a steelpan are experimentally studied. Modal analysis reveals the existence of numerous modes displaying internal resonances of order two and three, enabling the structure to transfer energy from low or high frequency modes. The complicated dynamics in forced vibrations is precisely measured by following the different harmonics of the responses. Energy transfers are explained in light of high order couplings, and a simple model displaying 1:2:2 internal resonance is fitted to the experiments. The measurements reveals that mode couplings are activated for very low amplitudes of order $1/25$ times the thickness, and that numerous modes are rapidly excited, giving rise to complex shape frequency response functions. This ease in transferring energy and coupling modes is a key feature for explaining the peculiar tone of the steelpan.

1. Introduction

Steelpanns are a tuned percussion instruments family coming from Trinidad and Tobago. They are made of oil barrels that are subjected to several stages of metal forming that stretch and bend the structure. The top of the barrel is pressed, hammered, punched and burnt in order to obtain a sort of main bowl within which convex substructures are formed. Each convex dome corresponds to a musical note, which natural frequency is precisely tuned according to harmonic relationships.

Depending on the instrument, on the selected note as well as on the tuner know-how, different harmonic relationships may be observed. In all cases, the second mode is tuned to twice the frequency of the fundamental, giving rise to 1:2 relationship. Due to the localization [2] it is generally observed that the second mode is degenerate so that a 1:2:2 internal resonance is present. The third mode is tuned either at the third or at the fourth of the fundamental frequency, so that 1:2:2:3 or 1:2:2:4 internal resonances are possible. Finally, higher modes are also found to be tuned, hence the presence of modes at six times and/or eight times the fundamental frequency, are also generally found [4, 2].

In normal playing where a note is stroke with a stick, vibrations amplitudes are such that geometric nonlinearities cannot be neglected, and is recognized as a key feature for explaining

the peculiar tone of the steelpan [1]. This nonlinearity combined with the numerous possible internal resonances, activates energy transfer to higher modes. In playing conditions, energy transfers up to a dozen of mode are usually observed.

The aim of this work is to study energy exchanges and activation of internal resonances in the nonlinear dynamics exhibited by the steelpan. Forced vibrations and frequency response curves are used to identify and localize instabilities and mode coupling. For that purpose, the different harmonics of the responses are followed during controlled step-by-step, forward and backward sweeps around the resonance of the first and the second eigenfrequencies. Frequency-response functions (FRFs) show complicated dynamics and activation of mode coupling from low to high-frequency. Scenarios of energy transfers are assumed, and checked versus comparisons of analytical FRFs obtained from multiple scale analysis of systems displaying 1:2:2 internal resonance. The study reveals that mode couplings are excited for vibration amplitude as small as $1/25$ the thickness, as well as the fact that numerous modes are involved in the vibration for a medium amplitude range of vibration.

2. Measurements

The steelpan shown in figure 1(a) is a right barrel of a *double second* (middle-high frequency steelpan). It is composed of 19 precisely tuned notes, distributed on three concentric circles, the lower notes being on the outer circle. This study is more particularly focused on G3 (of fundamental frequency f_1) and its harmonically tuned neighbours G4 ($2f_1$) and G5 ($4f_1$).

2.1 Modal analysis and linear characterization

Modal analysis is used to characterize the linear behaviour of the structure by identifying eigenfrequencies, mode shapes and modal damping coefficients. In the experiment, the steelpan is excited by a homemade non-contact coil/magnet exciter, for which the equivalent point force is estimated by recording the current intensity in the coil [5]. The steelpan vibratory response, in velocity, is measured with a laser vibrometer. Figure 1 shows the precise location of the excitation point E on the G3 note, and the measured transfer function in the frequency range $[0, 1700]$ Hz. One can see that the first three modes are perfectly tuned like f_1 , $2f_1$ and $4f_1$, while the fourth and the fifth depart a little from the perfect harmonic relationship, and are slightly shifted from the exact $6f_1$ and $8f_1$ relation. At $2f_1$, a double peak is clearly visible indicating that the mode is degenerate with two mode shapes around the same frequency.

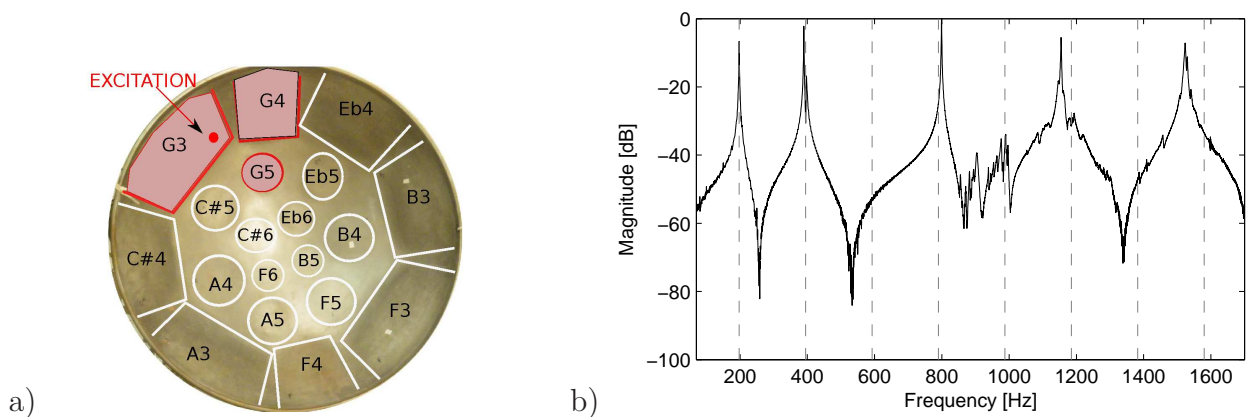


Figure 1. a) Modal analysis of the steelpan excited on the note G3. b) Associated frequency response function, measured at the excitation point E

The experimental set-up used for measuring FRFs is shown in figure 2(a), as well as the location of two selected measurement points, the first one (A) on the G3 note, the second one (B) on the G4 note (Fig. 2(b)). In the remainder of the paper, $w(x, t)$ denote the transverse displacement, and w_A, w_B refer to the displacement at points A and B, respectively. Linear measurements are performed by selecting a very small value of the excitation force.

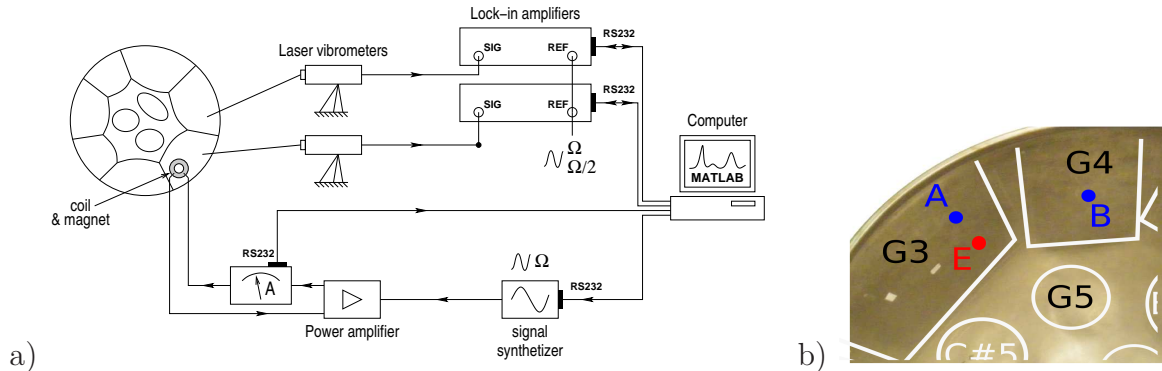


Figure 2. a) Experimental set-up. b) Location of points measurements on the steelpan's notes

Forward and backward frequency sweeps around the linear resonances are then measured and reported in figure 3. Selecting the excitation frequency f_{dr} is in the vicinity of f_1 allows a precise measurement of the fundamental frequency at $f_1=197.6$ Hz (Fig. 3(a)). The associated mode shape is, as awaited, located on the note G3 with a single maximum, as shown in figure 3(d). When $f_{dr} \simeq 2f_1$, two degenerate modes are identified. The first one (denoted mode 2) has for eigenfrequency $f_2=390$ Hz and its mode shape is composed of the second vibration pattern of the G3 note together with the fundamental vibration mode of the G4 note. The second one (denoted mode 3) has its eigenfrequency at $f_3=397.8$ Hz and its mode shape is similar except the fact that the pattern on the G4 note is out of phase (Fig. 3(b), 3(e), 3(f)). Finally, using the measurement at point B instead of the one at point E used for the transfer function in figure 1(b) reveals that at $4f_1$, two degenerate modes are also at hand, with eigenfrequencies $f_4=789.5$ Hz and $f_5=799.3$ Hz (Fig. 3), and mode shapes as shown in figure 3(g) and 3(h).

This linear analysis shows that almost harmonic relationships are present and the occurrence of 1:2:2:4:4 internal resonance is possible. More complicated scenarios invoking also the presence of the modes at $6f_1$ and $8f_1$ may also be activated in certain vibratory regimes. Forced vibrations at higher force amplitudes will now be detailed to depict how energy is transferred between these modes.

2.2 Nonlinear dynamics of the steelpan forced vibrations

In order to study the nonlinear behaviour of the steelpan, the amplitude of the excitation is increased. Two cases are investigated, where the driving frequency f_{dr} is selected either at the fundamental frequency, or in the vicinity of the second mode. The measured vibration obtained from the laser vibrometer at point A is decomposed in harmonic components ($f_{dr}, 2f_{dr}, 3f_{dr}, \dots$) by means of a lock-in amplifier (Fig. 2a). Frequency response curves are obtained by a step-by-step increasing and decreasing frequency sweep, where for each point, a settle time is awaited for the transient to die away, then forty measurement amplitudes are recorded in the steady state (Poincaré stroboscopy), in order to discriminate periodic and quasi-periodic regimes. Measurement of a forward sweep with the selected frequency range lasts 2 hours.

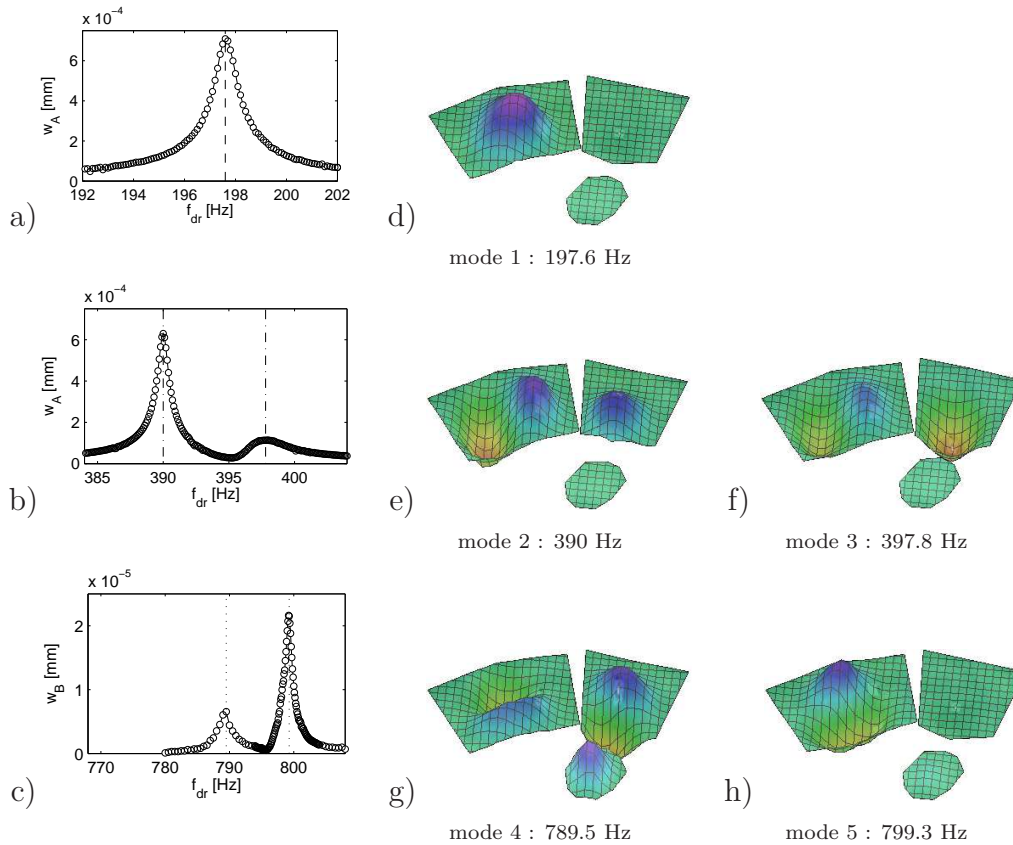


Figure 3. Displacement measurements for a low amplitude forced excitation ($I=0.01A$) and associated mode shapes around f_1 (---) ; $2f_1$ (- · -) and $4f_1$ (· · ·)

2.2.1 Low frequency excitation ($f_{dr} \simeq f_1$)

Two levels of excitation ($I=2A$ and $I=5A$) are shown in figure 4 to investigate the nonlinear dynamics of the steelpan in the vicinity of the first eigenfrequency at 197.6 Hz. Harmonics 1, 2 and 4 of the recorded displacement are shown, they are denoted respectively by w_{A1} , w_{A2} and w_{A4} .

For the first excitation amplitude shown ($I=2A$), the coupling between the first three modes is evident, as well as with the fourth mode at 789.5 Hz. Markers are inserted into the figures to precisely locate, in frequency, the different eigenfrequencies of the system. The complex shape of w_{A2} highlights the fact that the 1:2:2 internal resonance is already activated with a strong transfer of energy from the first to the second and third modes. Interestingly, this coupling is already effective for a vibration amplitude of the fundamental of 0.04 mm. As the thickness is 1 mm, one can conclude that the geometric nonlinearities effect are noticeable in the system response for vibration amplitudes of 1/25 times the thickness. Finally, w_{A4} participate to the response with a non-negligible amplitude which is not slaved to w_{A2} , with a strong peak in the vicinity of the fourth mode at 789.5 Hz. On the other hand, the second configuration at 799.3 Hz do not appear in the response so that one can assume that a 1:2:2:4 resonance is here activated.

For the second vibration amplitude ($I=5A$), the same resonance scenario seems to be at work, but the dynamics is now complicated with the appearance of a quasiperiodic regime in a frequency range around 193 Hz and with a slight difference between increasing and decreasing frequency experiments. The shape of the solution branches, as compared to theoretical ones of the 1:2 and 1:2:2 internal resonance that can be found in [3, 2], appears slightly different, as a reflection of the fact that the cubic nonlinearity is not anymore negligible. However, the 1:2

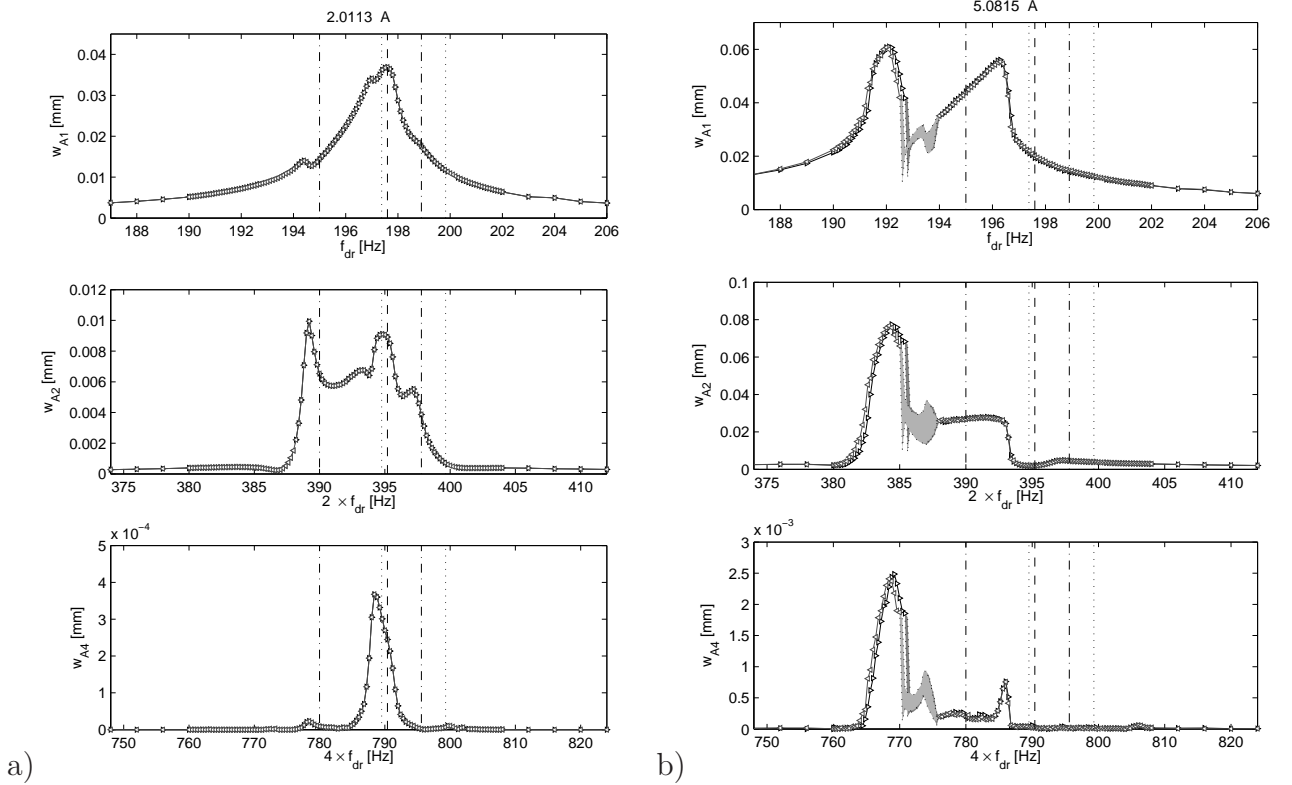


Figure 4. Frequency response curves for $I=2A$

(left column) and $I=5A$ (right column), excitation frequency f_{dr} in the vicinity of the first mode. Per lines: harmonics 1, 2 and 4 of the measured displacement w_A . Forward ($-\triangleright-$) and backward ($-\triangleleft-$) frequency sweeps. Linear eigenfrequency markers f_1 ($- -$) ; $2f_1$ ($- \cdot -$) and $4f_1$ ($\cdot \cdot \cdot$)

coupling between f_1 and f_2 seems to dominate the whole response, as indicated by the strong peak around 192 Hz, as compared to the other couplings. Finally the response maximum is now shifted to lower frequencies with this maximum amplitude at 192 Hz, which also advocates for the excitation of cubic nonlinear terms.

2.2.2 High frequency excitation ($f_{dr} \simeq 2f_1$)

In this section, the driving frequency f_{dr} is selected in the range [380, 405] Hz, where the degenerates modes with two configurations at 390 Hz and 397.8 Hz, are present (mode and 3). As in the previous section, two forcing amplitudes are shown, $I=2$ A and $I=5A$. The signal decomposition by harmonics is also perform to analyze mode couplings, but contrary to the previous case, the lock-in amplifiers are selected at half the driving frequency in order to recover the component oscillating at the fundamental frequency, around $f_{dr}/2$, in order to measure the coupling with the fundamental mode. In figure 5, w_{A1} thus refers to the component at $f_{dr}/2$ in the vibration, while w_{A2} denotes the component at the driving frequency. For the analysis, we also add in the figure the sixth harmonic w_{A1} (oscillating at three times the excitation frequency).

For the moderate amplitude of forcing ($I=2A$), one can observe that the 1:2 internal resonance between the first configuration (directly excited) at 390 Hz and the fundamental mode, is excited, giving rise to energy transfer and the occurrence of a component w_{A1} . On the other hand, the second configuration is not enough excited to activate the coupling with the fundamental, so that a 1:2 resonance scenario is here present. On the other hand, around the second configuration, a clear coupling with the modes at $4f_1$ is observed as denoted by the important response of w_{A4} . Here, a 1:2:2 scenario between the second configuration at 397.8 Hz, and the

two upper modes at 789.5 and 799.3 Hz, is observed, but it seems that the frequency range between these two energy transfers (1:2 and 1:2:2) is enough separated to assume two different coupled dynamics. Finally, w_{A6} shows an important peak at 1155 Hz which is not slave to other component, so that one can assume that cubic nonlinearities are already active for this range of amplitude and a small amount of energy is transferred via 1:3 resonance between w_{A2} and w_{A6} .

For the larger amplitude ($I=5A$), the two identified couplings are still present, with now a strong hysteresis in the response between forward and backward sweep in the first 1:2 resonance, leading to jum phenomena. Moreover, the frequency range of that 1:2 resonance now overlaps the frequency band where the coupling with w_{A6} is observed, so that a 1:2:6 scenario should be assumed. As in figure 4(b), the positions of the solution branches are slightly shifter to the low-frequencies, indicating a global softening behaviour and the excitation of cubic non-linearity.

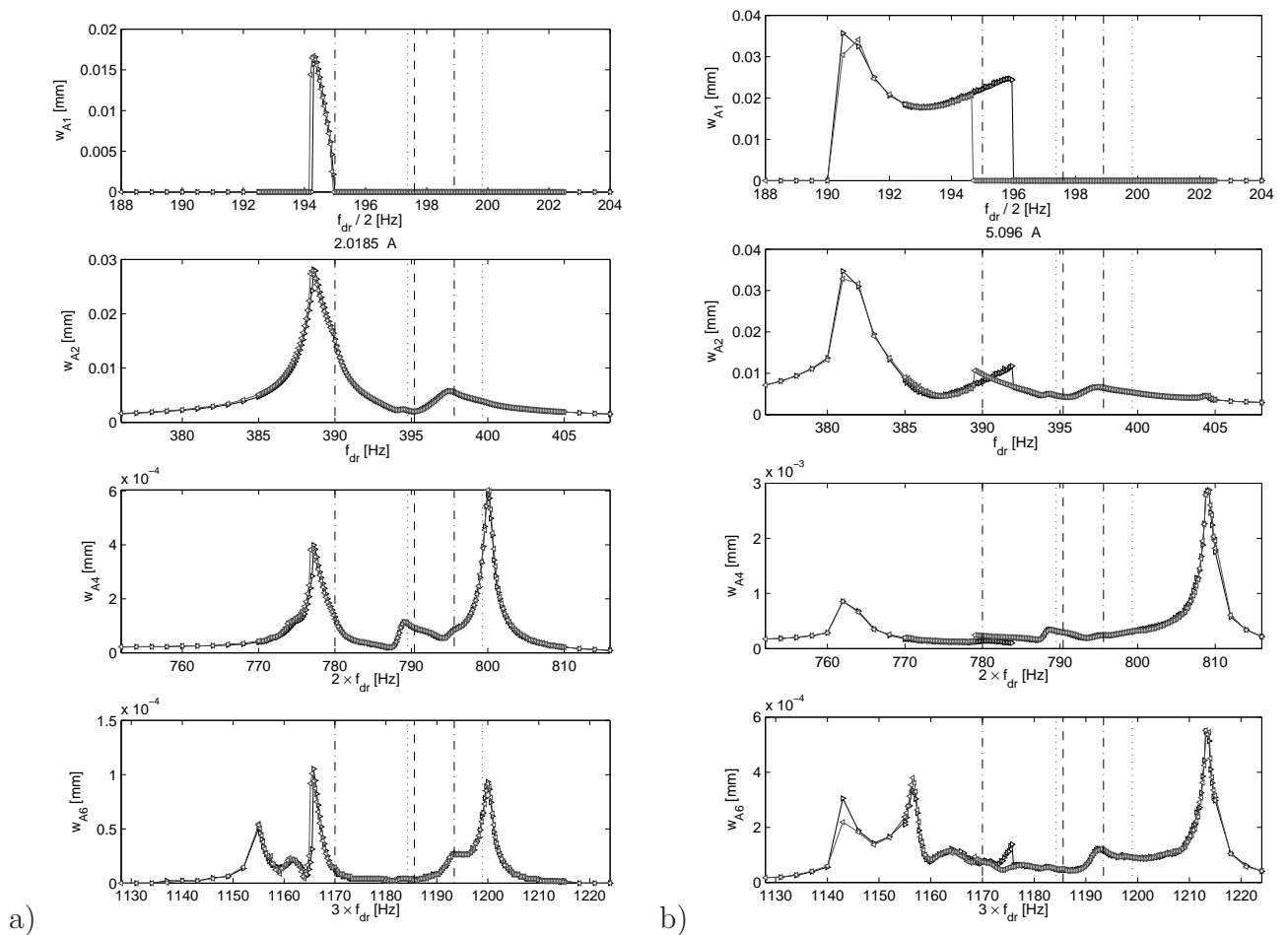


Figure 5. Frequency response curves for $I=2A$ (left column) and $I=5A$ (right column), excitation frequency f_{dr} in the vicinity of the second mode. First line: component at half the driving frequency. Second, third and fourth lines: component at the driving frequency f_{dr} , $2f_{dr}$ and $3f_{dr}$. Forward ($- \triangleright -$) and backward ($- \triangleleft -$) frequency sweeps. Linear eigenfrequency markers f_1 ($--$); $2f_1$ ($- \cdot -$) and $4f_1$ ($\cdot \cdot \cdot$)

3. Model fitting to experiment

In order to gain insight into the complicated dynamics exhibited by the steelpan in forced vibrations, a simple model involving three internally resonant modes presenting a 1:2:2 relationship, is used to fit the experimental FRFs. When the driving frequency is in the vicinity of the fundamental mode, analytical solutions are accessible via multiple scales analysis [2]. These will be used to fit the unknown nonlinear coupling coefficients and obtain a better identification of the energy transfers in the first case analysed in section 2.2.1 (Fig. 4). The three-modes model reads:

$$\ddot{q}_1 + \omega_1^2 q_1 = \varepsilon [-2\mu_1 \dot{q}_1 - \alpha_1 q_1 q_2 - \alpha_2 q_1 q_3 + F_1 \cos \Omega t] \quad (1a)$$

$$\ddot{q}_2 + \omega_2^2 q_2 = \varepsilon [-2\mu_2 \dot{q}_2 - \alpha_3 q_1^2] \quad (1b)$$

$$\ddot{q}_3 + \omega_3^2 q_3 = \varepsilon [-2\mu_3 \dot{q}_3 - \alpha_4 q_1^2] \quad (1c)$$

where $\Omega = 2\pi f_{\text{dr}}$, q_k denotes the modal amplitude of mode k , $\omega_k = 2\pi f_k$ its angular frequency and μ_k and μ_k its damping coefficient. These values are extracted from the linear analyses performed in section 2.1. Considering the first experiment, we have $\Omega \simeq \omega_1$, and $\omega_2 \simeq 2\omega_1$, $\omega_3 \simeq 2\omega_1$. The amplitude of the forcing is deduced from the measurement, and the nonlinear coupling coefficients α_i are fitted from experimental FRFs.

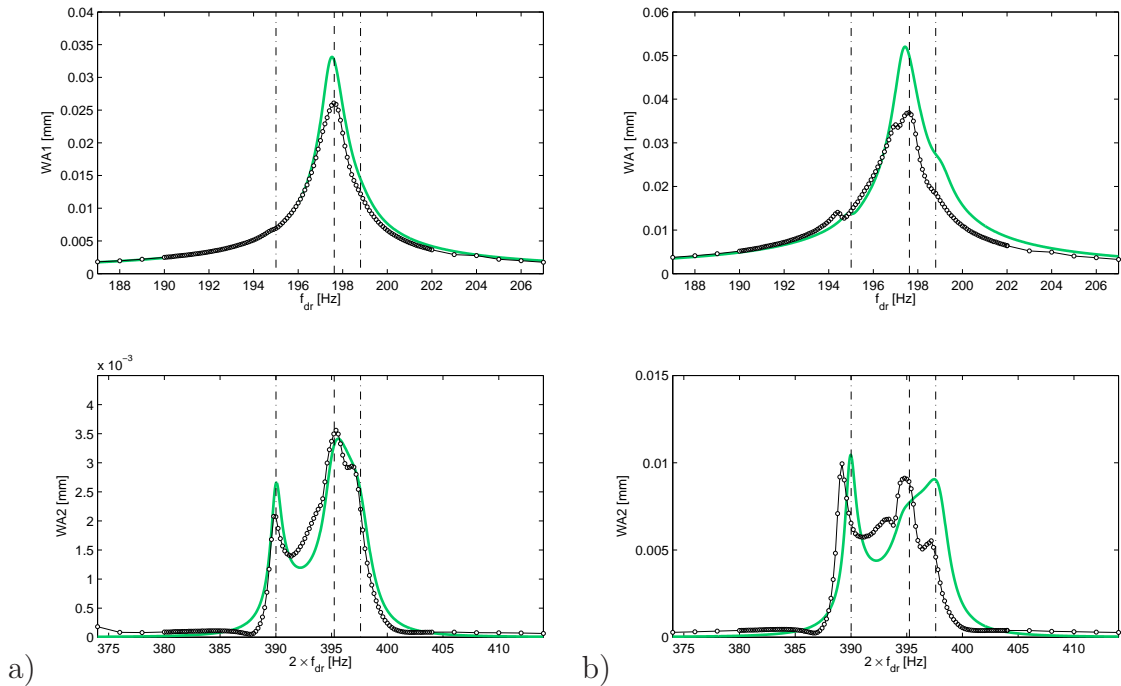


Figure 6. Experimental fitting by a 1:2:2 multiple scale model. Frequency response curves for $I=1A$ (left column) and $I=2A$ (right column), excitation frequency $f_{\text{dr}} \simeq f_1$. Harmonics 1 and 2 of the measured displacement w_A . Linear eigenfrequency markers f_1 (---) and $2f_1$ (-.-)

The best fit obtained is shown in figure 6, where two forcing amplitudes are represented, $I=1A$ (a smaller value as compared to the first case shown in figure 6) and $I=2A$ (corresponding to figure 6(a)). For the smaller amplitude ($I=1A$), one can see that the model fairly recovers all the features of the FRF. The discrepancies can be easily attributed to the non-modelled presence of the fourth mode. As noted in section 2.2.1, a 1:2:2:4 coupling is observable. Consequently, more energy is transferred from the directly excited mode that has to feed also the

fourth mode in the experiment. This explains the fact that the model overpredicts the maximum vibration amplitude of w_{A1} . Secondly, the presence of the mode at $4f_1$ also explains the more complicated shape of the FRF for w_{A1} in the vicinity of 396 Hz. The discrepancies are enhanced when increasing the forcing amplitude to $I=2A$. The overprediction of the maximum amplitude in w_{A1} is larger, as a reflection of the fact that more energy has been transferred to the non-modelled fourth mode, which response is more significant, contributing in the important peak around 395 Hz for w_{A2} .

This fitting shows that simple models can be used to enhance the comprehension of the complicated dynamics experimentally observed. The model displaying 1:2:2 internal resonance allows to recover the main feature of the FRFs, while non-modelled effect appears to be easily interpreted. The most complete model for that case should be a 1:2:2:4 one, unfortunately analytical solutions for that problem are not tractable. Further work will consider fitting models of 1:2 and 1:2:2 to the high frequency excitation shown in figure 5. Other notes of the steelpan are also investigated.

4. Conclusion

Nonlinear vibrations of one note of a steelpan have been experimentally investigated. Forced vibrations have been used to gain a better comprehension of the different mode couplings. The main features found by this experimental analysis are the following :

- Nonlinear coupling and energy exchange are excited for very small vibration amplitudes, of the order of 1/25 the thickness
- Numerous modes displaying internal resonances are present and excited, hence resulting in a complex, high-dimensional dynamics
- Fine analysis allows to isolate features of 1:2:2, 1:2:4 and 1:2:4:6 internal resonances
- Cubic nonlinearity appears not negligible for the usual amplitudes of vibrations encountered
- Simple models can be used to explain the most important features of the FRFs

In normal playing, all these features are simultaneously excited, giving rise to a rich tone with a build-up of frequency through energy transfers and a complicated dynamics involving from 3 to 10 modes, resulting in the peculiar and interesting sound of the steelpan.

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