Reduced-order models of MEMs including large rotations with normal form approach

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Abstract. The design of resonating micro electro-mechanical devices (MEMS) requires rapid and accurate estimation of its dynamical characteristics. In this framework, reduced order models (ROMs) are key for fast and accurate identification. A direct normal form computation on the Finite Element (FE) discretization is applied to a MEMS involving large rotations and severe geometrical nonlinearities. The ROM is compared with simulations of the complete structure using harmonic balance and continuation on the full-order FE model.

Introduction

This work addresses the development of reduced-order models of mechanical systems subjected to large rotations. The direct normal form method as proposed in [1, 2] is used for a MEMS structure subjected to large rotation amplitudes creating in turn important geometrical nonlinearity, and for which a reduction method based on implicit condensation and expansion failed due to the importance of the inertial term. The results provided by the ROM can be compared to full-order model simulation that are handled thanks to Harmonic Balance Finite Element Method (HBFEM), where a continuation method on all the degrees of freedom of the structure is implemented thanks to a Harmonic Balance assumption.

Results and discussion

The study is performed by applying the direct normal form to reduce the model of a structure excited in its torsional mode. A representation of the structure and of the mode under investigation is shown in fig. 1a. The resulting single degree-of-freedom model is expressed as:

$$\ddot{R} + 2\xi S + \omega_p^2 R + (A + h)R^3 + BRS^2 + 2\xi BR^2 S = F_p \cos(\omega t) \tag{1}$$

with R the *normal* coordinate, S its velocity, ξ a damping coefficient, ω_p the eigenfrequency, F_p the forcing amplitude, and A, h, and B coefficients derived from the reduction procedure. The normal coordinates describes the dynamics in an invariant-based span of the phase space, such that Eq. (1) represents the reduced dynamics in the nonlinear normal mode (NNM) associated to the linear mode of interest. The NNM is an invariant manifold that is tangent at origin to its linear counterpart.

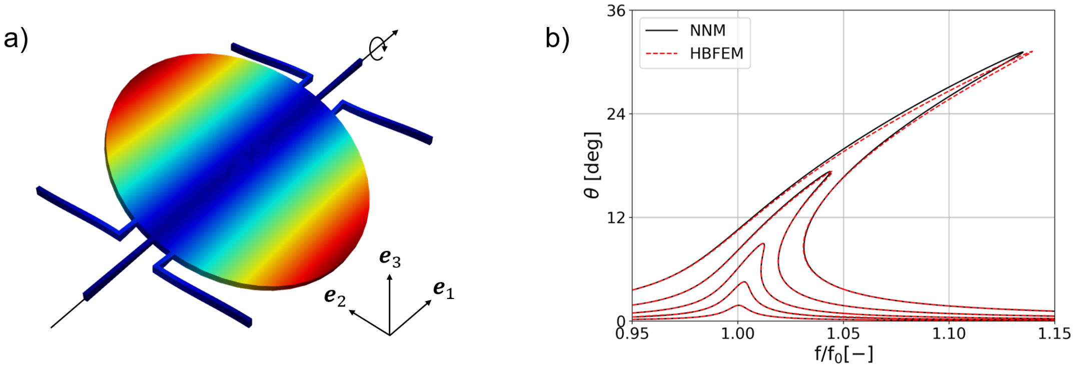

Figure 1: (a) Eigenmode shape of the test structure, (b) comparison between NNM and the reference HBFEM solution.

The frequency response function derived by the normal form approach is compared in fig. 1(b) to the solution computed from the full-order model using the HBFEM with numerical continuation. The comparison shows an excellent agreement between the two methods. The dimensionality of the full-order model is reduced from ten thousands degrees of freedom to one, with negligible loss of accuracy for most applications.

References

[1] A. Vizzaccaro, Y. Shen, L. Salles and C. Touzé: Model order reduction methods based on normal form for geometrically nonlinear structures: a direct approach, *Proceedings of ENOC 2020*, European Nonlinear Dynamics Conference, Lyon, France, July 2020 postponed to July 11-16, 2021.

[2] A. Vizzaccaro, Y. Shen, L. Salles, J. Blahos and C. Touzé (in preparation for *CMAME*) Direct computation of normal form for reduced-order models of finite element nonlinear structures.