NONLINEAR AXISYMMETRIC VIBRATIONS OF GONGS

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Abstract

In this paper, experiments on a large vibrating gong, or "Chinese tam-tam" are reported and analyzed. Various tools are used in order to determine with precision the typical scenarii occurring in such an instrument as the magnitude of the flexural displacement increases from linear to moderately nonlinear regime, in the case of harmonic forced excitation. Variations of the excitation frequency at constant magnitude are also examined. The results clearly show the relative contribution of some particular axisymmetric and asymmetric modes in the nonlinear behavior of the gong. These experimental results are aimed at providing a reference basis for current and future theoretical studies (see, for example, [1]).

INTRODUCTION

At a previous ISMA Conference, preliminary results on the nonlinear forced vibrations of cymbals were presented by the two first authors. This paper showed qualitatively that cymbal spectra exhibit combination of resonances. However, the main part of the paper was mostly devoted to signal processing tools with the intention of determining the number of active degrees of freedom in the chaotic regime [2]. The present paper has a close filiation with the previous one, in the sense that its ultimate goal is to shed more light on the acoustics and vibrations of nonlinear percussive instruments. However, the main differences with the previous paper are the following:

- The present instrument is a gong, whose geometry and boundary conditions, among other things, differ significantly from those of a cymbal.
- The present study concentrates on weak to moderate nonlinearity rather than on chaos.
- The use of signal processing analysis is limited to time and spectral analysis. However, this analysis is complemented by spatial informations provided by finite element modeling and modal analysis.

EXPERIMENTS

The experiments consist of driving the gong by means of a non-contacting electrodynamical device at a frequency close to one of its particular mode of vibration (see [3]). The excitation is provided by a cylindric coil driven by a high power amplifier, a small magnet being glued on the gong. The distance between the end of the coil and the gong is of the order of 1 mm. In most experiments, the magnet is glued at the center of the gong and a driving frequency equal to 556 Hz is selected, which corresponds to the eigenfrequency of the (0,3) axisymmetric mode (see Table I). The velocity and acceleration of one selected point of the gong are measured simultaneously by means of a laser vibrometer and of an accelerometer, respectively (see Fig. 1). In a first series of experiments (**Experiment 1**), the driving force increases gradually from 0.05 to 5 N, at

Figure 1: *Experimental set-up*

constant frequency. In a second series of experiments (**Experiment 2**), the driving force is kept constant and equal to 2.5 N, while the frequency varies in a limited range. In each situation, both velocity and acceleration signals are continuously recorded on a DAT tape for further processing.

RESULTS

For **Experiment 1**, Fig. 2 shows the spectrogram of the acceleration signal over the 90 s sequence of the excitation. This spectral analysis shows a typical behavior of quadratic nonlinearity. For the sake of clarity, only the low-frequency range of the analysis is presented here. Accurate analysis shows that the relative difference (in dB) between the fundamental (556 Hz) and its second harmonic (1112 Hz) increases from -47 dB to -2 dB. Around 80 s, the magnitude of the driving force is roughly equal to 1 N. At this time, one can see the apparition of combination resonances whose frequencies correspond to other modes of the gong (see Table I). Most of these modes are asymmetric and have a node at the center, so that their existence can only be due to nonlinear effects.

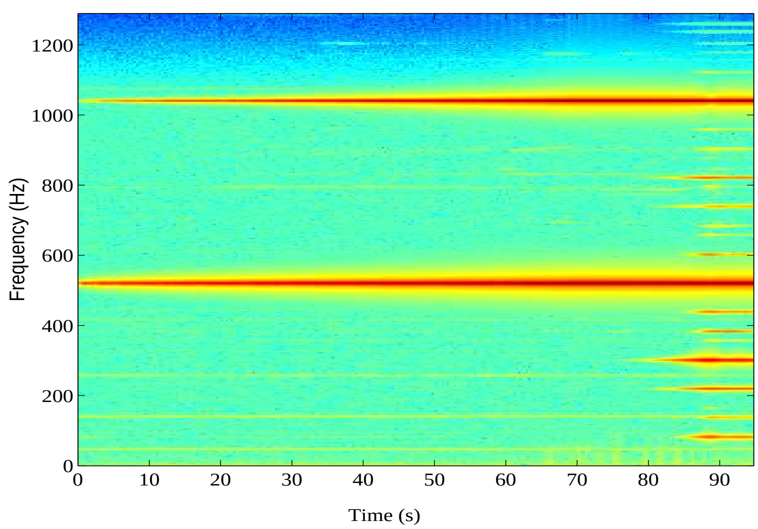

Figure 2: *Acceleration vs time. Increasing force.* F_{exc} *= 556 Hz. Quadratic nonlinearity and apparition of nonlinear resonances.*

Calculated Frequency (Hz)	Measured Frequency (Hz)	Mode Number	Analog Type	Symmetry Properties
79	87	(0,1)	Clamped plate	S
142.5	147/132	(1,1)	Clamped plate	AS
167.7	175	(3,0)	Edge mode	AS
225.5	223/235	(2,1)	Clamped plate	AS
267.7	275	(0,2)	Clamped plate	S
337.6	322/314	(3,1)	Clamped plate	AS
350	380	(4,0)	Edge mode	AS
369	410	(1,2)	Clamped plate	AS
472	468	(4,1)	Clamped plate	AS
497.6	498	(2,2)	Clamped plate	AS
518.5	556	(0,3)	Clamped plate	S

Table I: *Properties of the modes involved in the nonlinear regime.*

The effect of quadratic nonlinearity is also seen on the acceleration waveforms shown in Fig. 3. These waveforms show that the distorsion of the signal can be viewed as the result of nonlinear stiffness, the gong being relatively stiffer in one direction of the transverse motion than in the other, which is a typical effect of curvature [1]. This time history is similar to comparable results observed in nonlinear vibrations of shells [4]. For time > 80 s, in **Experiment 1** shown in Fig. 2,

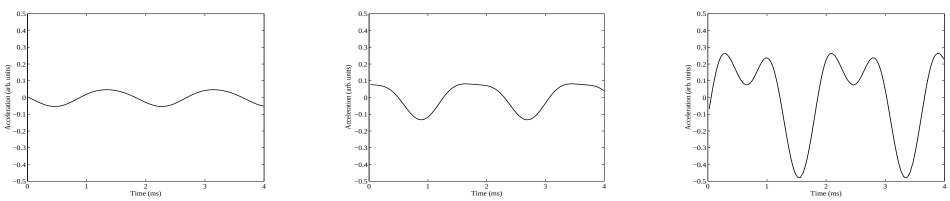

Figure 3: *Acceleration waveforms. Left: at time t= 10 s. Center: t= 40 s. Right: 70 s.*

the apparition of new resonances is determined by specific relationships between the frequencies. This corresponds to a general property of nonlinear systems with several degrees of freedom [5]. In the present case, where the excitation frequency is $F_{exc} = 556$ Hz, the typical scenario, for frequencies lower than F_{exc}, is the following:

1. Apparition of resonances at 322 Hz and 235 Hz, which correspond to the asymmetric (3,1)- and (2,1)-mode of the linear regime. The corresponding modal shapes are shown in Fig. 4 and Fig. 5.

2. With increasing amplitude, the spectral analysis shows simultaneous apparition of combined resonances at 87 Hz, (0,1)-mode, and 468 Hz, (4,1)-mode. This apparition is rapidly followed by two other combined resonances at 147 Hz, (1,1)-mode, and 410 Hz, (1,2)-mode.

3. Only at very high excitation level, not visible on Fig. 2, one can notice the apparition of combined resonances at 175 Hz, (3,0)-mode, and 380 Hz, (4,0)-mode.

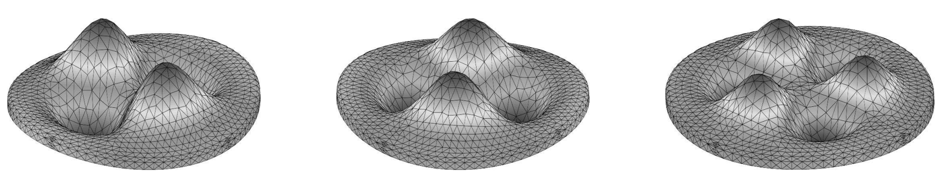

Figure 4: *Examples of asymmetric modes involved in nonlinear axisymmetric forced vibrations of the gong. Finite element modeling. Left: (2-1)-mode ; first configuration . Center: (2-1)-mode ; second configuration. Right: (3,1)-mode.*

Figure 5: *Experimental modal analysis of the asymmetric modes presented in Figure 4.*

At each stage of this experiment, one can see that the frequencies of the combination resonances are linked together by the relation:

$$322 + 235 \;\approx\; 87 + 468 \;\approx\; 147 + 410 \;\approx\; 175 + 380 \;\approx\; 556 \pm 1 \; Hz \tag{1}$$

which agrees with theoretical studies which predict that, in the case of quadratic nonlinearity, we must have

$$F_{exc} \approx f_n \pm f_m \tag{2}$$

where f_n and f_m are eigenfrequencies of the system [5]. In coherence with this general rule, it has been observed experimentally that exciting the gong at high level in the same manner around 275 Hz, (0,2)-mode, or around 87 Hz, (0,1)-mode, does not yield combination of resonances, since there are no modes, below these frequencies, for which the relationship expressed in Eq. (2) can be satisfied. Notice in passing that, at the time where combination resonances just appear, it is observed that, the magnitude of the second harmonic decreases slightly, which can be viewed as an energy transfer.

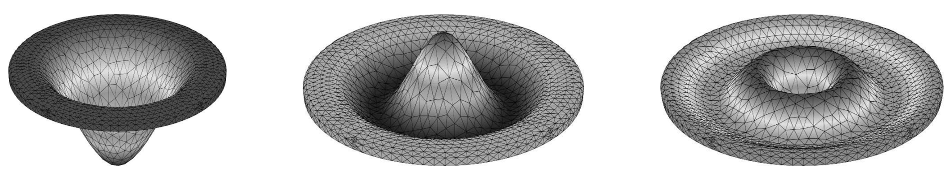

Figure 6: *First three axisymmetric modes of the gong. Left: (0,1) mode at 87 Hz, center: (0,2) mode at 275 Hz, right: (0,3) mode at 556 Hz.*

For higher frequencies, between F_{exc} and 2 F_{exc}, for example, it has been observed that the combination of resonances obey to the same relation than in Eq. (2), with the additional feature that the combinations now can involve the harmonics of the lowest modes, which contributes to complexify the analysis of all frequencies. These results show in detail how the nonlinear process of modes combination are responsible for the progressive enrichment of the spectrum with increasing amplitude of the excitation. The order of apparition is probably due to differences in nonlinear coupling coefficients, whose determination needs further theoretical studies.

Observation of the modal shapes of the gong shows that most of the involved modes look similar to clamped plates modes. This is the case for almost all modes shown in Table I, if we except the (3,0) and (4,0) modes which can be qualified as “edge modes” (see Fig. 7). In general, **Experiment 1** shows that coupling exists between axisymmetric and asymmetric modes, which confirms theoretical studies made by the authors where it has been shown that the coupling is mainly due to membrane effects [1], [6]. Intuitively, by examining the modal shapes, one can reasonably understand that the degree of mechanical coupling between the axisymmetric (0,3) mode and these edge modes is weaker than with the other asymmetric modes. However, this hypothesis needs further theoretical confirmation. This experiment also confirms that the selection of the modes involved in the coupling obeys to specific relations between their frequencies.

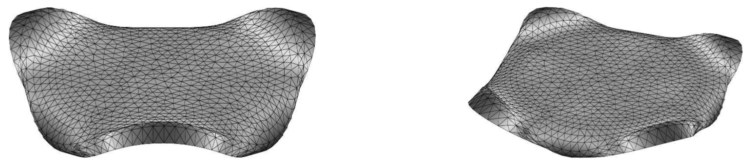

Figure 7: *Examples of "edge modes". Left: (3,0) mode at 175 Hz. Right: (4,0) mode at 380 Hz.*

The gong, like plates [3,6], shows two distinct configurations for the asymmetric modes, as seen in Fig. 4. For a completely homogeneous and isotropic structure, both modal shapes have the same eigenfrequency. For the present instrument, Table I shows that some of these frequencies are clearly not identical (modes (1,1), (2,1) and (3,1)) which is an indication that the gong is either inhomogeneous or/and anisotropic. At this point, it is interesting to notice that the consequence of these measured differences in frequency, for the same modal shape, is to allow the conditions in frequency presented in Eq. (1). The crucial point of whether or not this particularity is the result of a controlled strategy by the maker is not yet elucidated.

Still whithin the general framework of **Experiment 1**, the magnet is now glued a few cms off the axis of the gong in order to excite asymmetric modes with progressively increasing force. With $F_{exc} = 235$ Hz, the nonlinear vibrations show apparition of frequencies at 87 and 147 Hz, respectively. With $F_{exc} = 322$ Hz, combination resonances at 235 and 87 Hz are measured. In both cases, one can see that the general condition expressed in Eq. (2) is fulfilled, since we have:

$$87 + 147 \approx 235 \pm 1Hz \quad \text{and} \quad 235 + 87 \approx 322 \pm 1Hz \tag{3}$$

Finally, in **Experiment 2**, measurements are made at constant force and varying driving frequency. Resonance curves, which show the magnitude of the acceleration as a function of excitation frequency, are obtained. The shape of such curves are similar to those corresponding to a modified Duffing equation, with quadratic terms added [5]. Due to the fact that the resonance around the (0,3)-mode at 556 Hz is of the "softening type", the maximum of the resonance curve is obtained for frequencies lower than 556 Hz (typically 553 to 554 Hz, for the range of force used in our experiments). One can see, in this case, that the conditions of the experiments lead to the apparition of a subharmonic of order 2, since we have:

$$554/2 = 277 \ Hz \approx 275 \ Hz \tag{4}$$

This feature is illustrated in Fig. 8 which shows the spectrogram of the acceleration, for F_{exc} = 554 Hz. Around time t=45 s in the sequence, one can clearly see the apparition of period doubling. In the present experiments, accurate measurements shows that the relative level of subharmonics is of the order of -40 dB, compared to the excitation frequency. Here, the (0,2)-mode is involved in the nonlinear coupling, which was not the case in the **Experiment 1**. The main purpose of this experiment is to illustrate to what extent the scenario in the nonlinear regime is dependent on the time history of the experiments: increasing the force to moderate level and then decreasing the frequency will not yield the same bifurcation than increasing the force at constant frequency.

SUMMARY AND CONCLUSION

The main results of this experimental study on a Chinese tam-tam, which complements previous work made by other authors on this class of instruments (see [7], [8]), are the following:

1. The nonlinear phenomena observed in the vibrations of such an instrument have the character of quadratic nonlinearity.

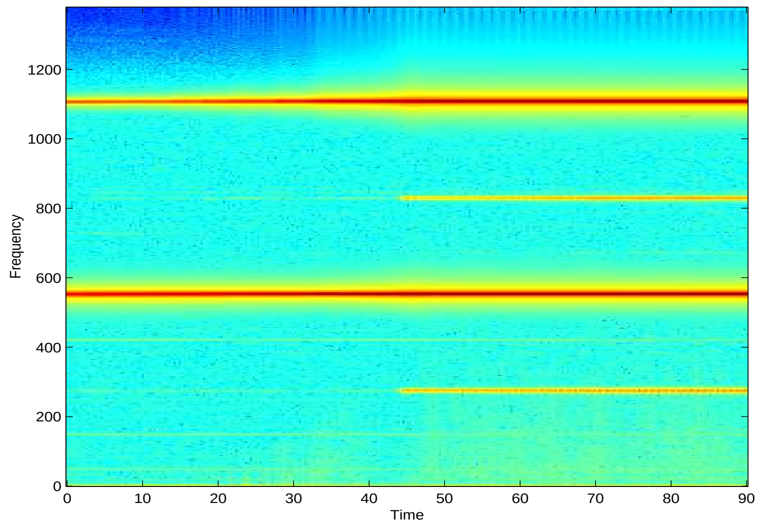

Figure 8: *Acceleration vs time. Constant force. Excitation frequency F_{exc} around 554 Hz. Quadratic nonlinearity and apparition of subharmonics.*

2. Forced excitation at sufficiently large amplitude, at a frequency F_{exc} close to one mode, leads to a bifurcation with apparition of lower frequencies, corresponding to other modes, provided that particular relations, such as the one expressed in Eq. (2), exist between those frequencies.

3. The properties of the bifurcations depend on the conditions of excitation. In our particular case, variations of excitation frequency at constant force, for example, yields subharmonics which were not observed at constant excitation frequency.

The goal of a theoretical work in progress now consists in justifying the various degrees of coupling coefficients between the modes in the nonlinear regime. Experiments are conducted in parallel on specially designed homogeneous shallow spherical shells. One expected result is to check whether or not nonlinear vibrations of these ideal structures show comparable frequency relationships such as those observed in gongs.

REFERENCES

[1] Thomas, O., Touzé, C. and Chaigne, A.,"Nonlinear behavior of gongs through the dynamics of simple rods systems", Proc. International Symposium on Musical Acoustics (ISMA2001), Perugia, Italy, 2001.

[2] Touzé, C., Chaigne, A., Rossing, T.D. and Schedin, S., "Analysis of cymbal vibrations using nonlinear signal processing methods", Proc. International Symposium on Musical Acoustics (ISMA98), Leavenworth, WA, USA, pp. 377-382.

[3] Thomas, O., Touzé, C. and Chaigne, A.,"Asymmetric non-linear forced vibrations of circular plates, Part II: Experiments", J. Sound Vib. (submitted), 2001.

[4] Leissa, A. W., *Vibrations of shells*, Acoustical Society of America, 1993 (Original issued by NASA, 1973).

[5] Nayfeh, A. H., Mook, D. T., *Nonlinear oscillations*, Wiley-Interscience, New York, 1979.

[6] Touzé, C., Thomas, O. and Chaigne, A.,"Asymmetric non-linear forced vibrations of circular plates, Part I: Theory", J. Sound Vib. (submitted), 2001.

[7] Legge, K. A., Fletcher, N. H., "Nonlinearity, chaos and the sound of shallow gongs", J. Acoust. Soc. Am., **86**(6), pp. 2439-2443, 1989.

[8] Rossing, T.D., Fletcher, N.H., "Nonlinear vibrations in plates and gongs", J. Acoust. Soc. Am., **73**(1), pp. 345-351, 1983.